THE

MECHANICS' CALCULATOR,

OR

WORKMAN'S MEMORIAL BOOK;

COMPREHENDING

PRINCIPLES, RULES, AND TABLES

IN THE VARIOUS DEPARTMENTS OF

MATHEMATICS AND MECHANICS;

USEFUL TO

MILLWRIGHTS, ENGINEERS, AND ARTISANS IN GENERAL.

WITH

Four Lithographic Plates.

BY WILLIAM GRIER.

How have we obtained this great superiority over these poor savages? Because science has been at work for many centuries, to diminish the amount of our mental labour, by teaching us the easiest modes of calculation.

Results of Machinery.

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TO

THOMAS CHALMERS, D. D.

OF THE UNIVERSITY OF EDINBURGH,

WHO HAS EVER BEEN ZEALOUS IN THE

DIFFUSION OF USEFUL KNOWLEDGE AMONG THE WORKING CLASSES,

AND A WARM ADVOCATE FOR

EVERY THING WHICH COULD CONTRIBUTE TO THEIR WELFARE,

THIS WORK IS DEDICATED,

WITH THE MOST SINCERE FEELINGS OF RESPECT,

BY THE AUTHOR.

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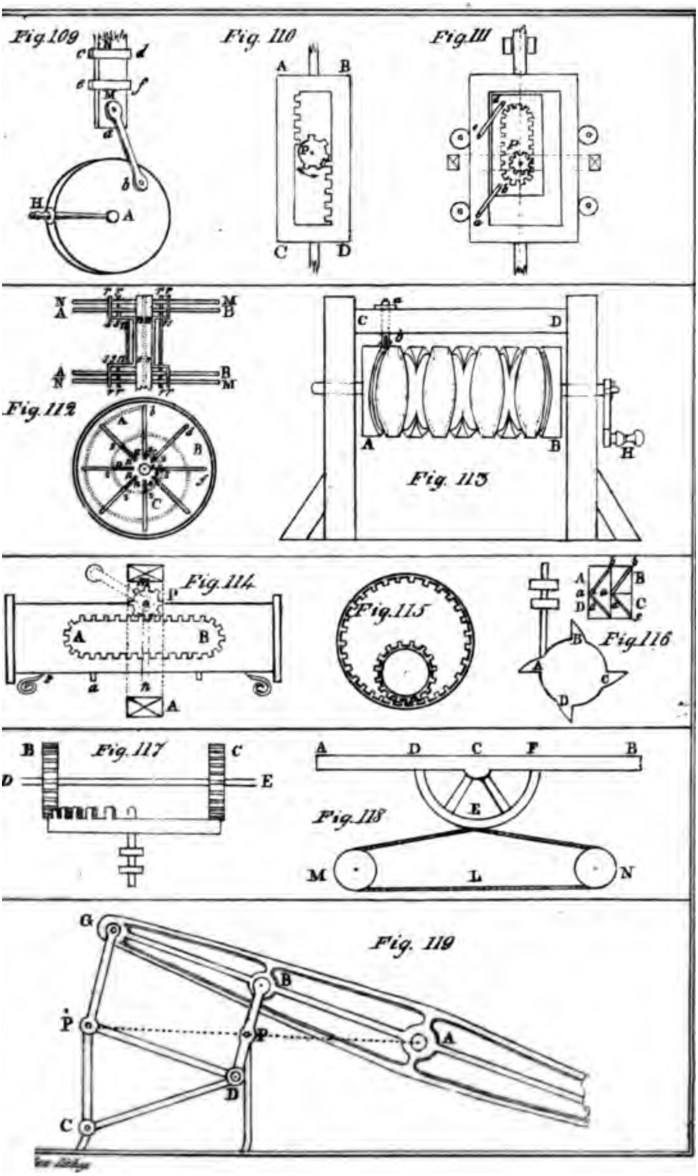
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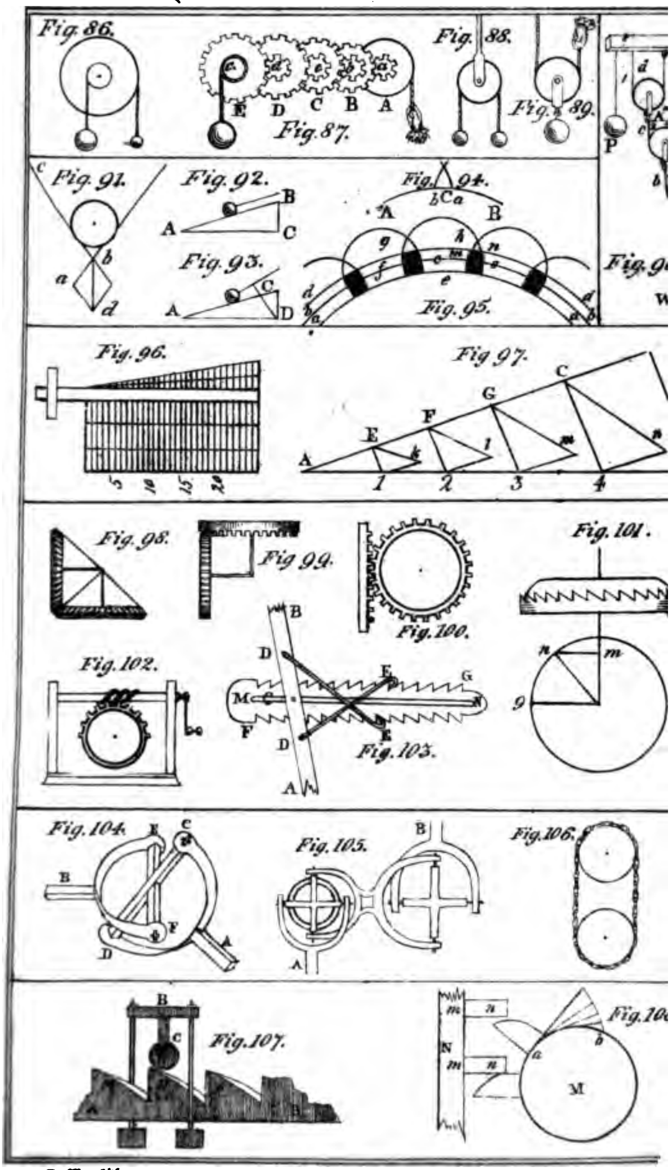
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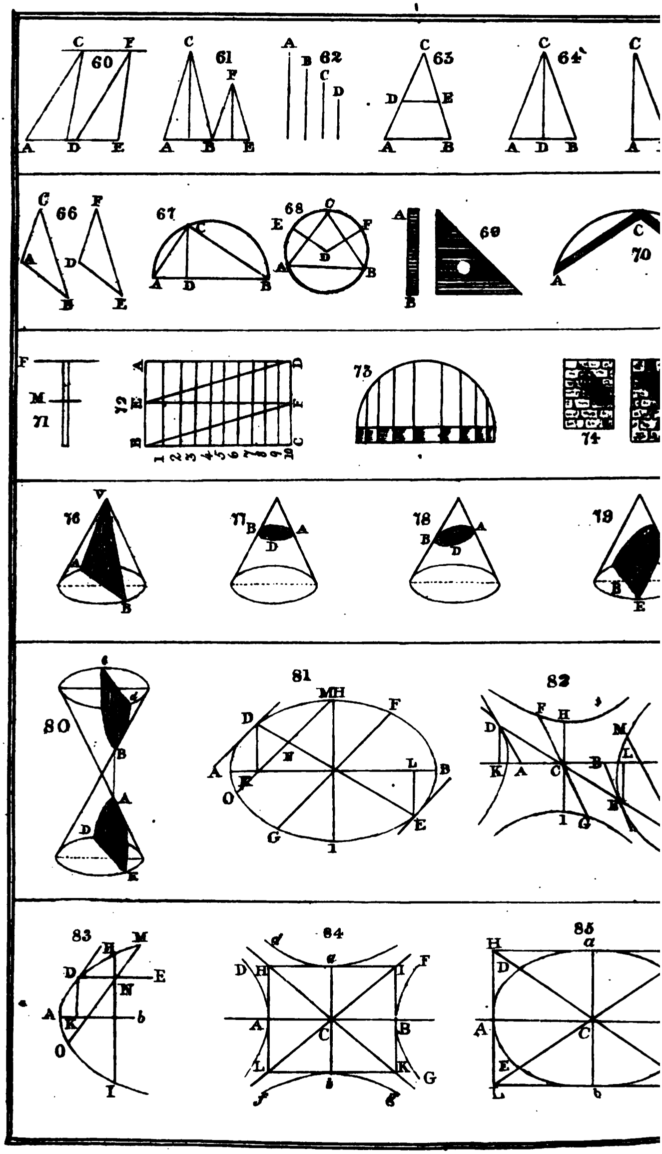


HYDROSTATICS.

HYDRODYNAMICS.

PNEUMATICS.

INTRODUCTION.

It is our intention in these introductory pages, to make a few observations on the nature of scientific knowledge, which may be useful to the young reader in enabling him to understand more clearly the subjects contained in the volume, and in guarding him against the adoption of false theory, or the wasting of his time in inquiries which can terminate in no useful result. Such introductory observations are rendered the more necessary, as a correct knowledge of the subjects to which they relate, is the only sure foundation on which there can be raised, a solid superstructure of science.

It is a general opinion that scientific knowledge is entirely different from all other kinds of knowledge; and that it requires for its cultivation a constitution of mind only to be met with here and there in the great family of mankind, and what is said of the poet is also thought of the philosopher—that he is *born*, *not made*. Although it must be confessed, that there have been few men who have made great advances in science, yet it is nevertheless true, that there are few men who can-

not make some advances. All men are certainly not equally endowed with capacities for the acquisition of scientific knowledge, but there are few men indeed who are totally unprivileged. The man who would relinquish scientific pursuits, merely because he had no hope of reaching the eminence of a Newton, a Watt, or a Davy, is no better than him, who, in despair of ever obtaining a share of wealth equal to that of the rich inheritor of the land, would cease to make any exertion to raise himself from a state of the most squalid wretchedness. We would not be understood, however, to bring the acquisition of knowledge into invidious comparison with the acquisition of wealth—the one is in every case a godlike employment, but the other is often the concomitant of vice.

The young mechanic should be made well aware that the knowledge of the man of science differs from the knowledge of the rest of mankind, not so much in kind as in degree; and the knowledge which guides the little boy in the erection of his summer-house constitutes a part of the knowledge which guides the best architect in the erection of the most splendid edifice. The boy raises his paper kite in the air, with no other end in view save his own amusement—he has learned to do so by seeing other boys do the same, and by trials he finds that the kite will fly better in a moderate wind than in a perfect calm, and that the weight at the tail may be too heavy or too light, and he regulates his actions accordingly: so far he is a little philosopher. A man raises a kite knowing all that the boy knew, but he raises his kite with a view of determining the state of

the atmosphere so far as electricity is concerned, for which purpose, instead of employing the hempen cord, which was sufficient for the purpose of the boy, he employs a metallic wire which he knows by experience will answer his design. In this respect the knowledge of the man is more extensive than that of the boy, but this additional knowledge has been obtained exactly in the same way as the knowledge of the boy, that is to say, by experience. The Indian, unlearned as he seems to be, is in some respects a philosopher. He sees daily that the paddle of his canoe is to appearance broken when he puts it into the water; but it is only to appearance, for by repeated trials, he finds that the paddle is as whole when in the water as when out of it. He knows also, by repeated trials, that the fish, while it shoots along through the clear flood, does not appear to be where it really is; for although he may be the most unerring of all marksmen, yet if he throws his dart directly at the fish he will certainly miss it. In vain will he try to strike the fish on the same principles as he strikes the bird flying in the air, but he finds, that when he directs his dart to a line which is nearer him than that in which the fish seems to move, he will strike the fish. The Indian remembers the circumstance of his paddle, and other circumstances of a like kind, and concludes, that, when bodies are seen through water they do not seem to be in the place in which they really are. When he knows and acts upon this principle, he is a man of science so far as this is concerned. The man of science, indeed, as we commonly understand that appellation, knows much more than this: he knows that many other substances

have a like effect in changing the apparent position of objects when seen through them, that one produces a greater and another a less change, &c. which knowledge is obtained in the same way as that of the Indian, but is more extensive.

An examination of facts is the foundation of all true science; but science does not consist in a mere examination of facts. They must be compared with each other, and the general circumstance of their agreement carefully marked. When we have compared several facts together, and find that there is one general circumstance in which they agree, this one circumstance becomes, as it were, a chain by which they are all linked together. This general circumstance of agreement, when expressed in language, is what is called a law. For instance, it is a law that all bodies, when left to fall freely, will tend to the earth, and this law has been framed by us, because in all cases which we have examined this has been the case; and the term gravity is nothing else than a name invented to express a circumstance in which we have found innumerable facts to agree. A collection of such laws which refers to some particular class of objects, when properly arranged, becomes what is called a theory. Thus, we see that a theory, properly so called, is founded on an examination of particular facts, and of course cannot refer to any other but those facts which have been examined; or, if it is attempted so to do, it is no longer a theory but a hypothesis or supposition.

In the examination of facts, it is to be observed, that we must depend on the information derived through the medium of the five senses, that is, the senses of seeing—

hearing—touching—tasting—and smelling;—for it is only by bodies affecting those organs that properties of matter become known to us; and all that the mind does is to compare and classify the information thus derived.

Our limits will not allow us to say more on this subject; we trust we have said sufficient to enable the young mechanic to see the truth of the assertion with which we set out; and the remainder of this introduction will be devoted to the removal of one or two errors regarding the utility of scientific knowledge, and a few directions to the reader on the further prosecution of his studies.

It is a common error to suppose that many of our greatest inventions and discoveries were made by accident. Many wonderful anecdotes are told in support of this assertion; but the very circumstance of their exciting our wonder is sufficient to show that they are out of the common course of our experience, and that, therefore, before they are received, they ought to undergo a careful examination. A multitude of facts might be adduced to prove that knowledge is more regularly progressive than is commonly imagined. Far be it from us to detract from the merit of those great men who have, from time to time, benefited mankind by their important discoveries; but from a survey of the history of science, we are led to the conviction, that wherever a new path has been struck out in the great field of truth, that path has been previously prepared by former inquirers. Had Kepler not discovered the three fundamental laws of the planetary motions, it is highly probable that the Principia of Newton never would have

issued from the pen of that illustrious man; and had it not been for the brilliant discoveries of Dr Black on the subject of heat, it is probable that Watt never would have made his improvements on the steam engine, that invaluable distributer of power. It is not unlikely, however, from the state of knowledge in the days of Newton, that, independent of the exertions of his mighty mind, the knowledge contained in the Principia would soon after have been given to the world by some one or more individuals—and the like may be said of the inventions of James Watt.

The great lesson which we would wish the young mechanic to learn from those observations is—that great discoveries are never made without preparation—that previous knowledge is necessary to turn what is called accidental occurrences to good account. And when he is told that the law of gravitation was suggested to Newton by the falling of an apple from a tree in his garden; or that the invention of the cotton jenny was suggested to Arkwright by the circumstance of a common spinning-wheel continuing its ordinary motion while in a state of falling to the ground—let him be well assured, that, had the minds of Newton and Arkwright not been previously stored with knowledge, these discoveries never would have been made by them. Apples and spinning wheels had fallen a thousand and a thousand times, but the knowledge necessary to turn these circumstances to good account, was first concentrated in the minds of these two illustrious benefactors of mankind.

While we are on this subject we cannot pass over

another very common prejudice, which we conceive has a very hurtful tendency on the progress of the young mechanic. We allude to the pride that some men take in boasting that all their knowledge is original; or that they are self-taught. This is, in other words, stating, that no assistance has been taken either from teachers or books; and goes only to prove, that the knowledge of the individual so circumstanced must be very limited indeed. The unassisted exertions of one man must be feeble indeed, when compared with the collected exertions of the many who have gone before him in the career of discovery. That man must know little of geometry, who has not availed himself of the use of Euclid's Elements, or some work of a similar nature; and the Elements of Euclid would have been meagre and confined, had he not availed himself of the discoveries of his contemporaries and predecessors. A like remark may be made on the cultivation of every department of knowledge; and to those whom we are now addressing we say—learn from others all that you possibly can, and when you have done so, try to correct and improve what you have obtained. We know of no dishonourable means of acquiring knowledge, and therefore wherever we meet it we are disposed to respect it, even though it should not contain one particle of originality, if such be possible; for it is not easy to conceive how any man should be in possession of useful knowledge, and not make some new application of it; and a new application of an old principle is certainly one constituent of originality. With a knowledge of what others have done, that workman will be less likely to

waste his time in enterprises which may ruin him by their failure or in speculations which are unsupported by the principles of science.

In the museum of the mechanics class of the university founded by the venerable Anderson of Glasgow, there is preserved the model of a contrivance to procure a perpetual motion. For the contrivance and execution of this beautiful specimen, we are, we believe, indebted to an ingenious clock-maker of Dundee, who has proven himself a master in the use of his tools. But had he been acquainted with the first principles of mechanics, or with the nature and failure of the various attempts which had been made before his time for the same purpose, he would have seen the utter folly of his enterprise, and would have spent the seven years which he occupied in the construction of this truly beautiful model in some more useful employment. These seven years might have been employed in constructing time-pieces which would have been of infinite service to the commerce and navigation of his country—in guiding the lonely mariner when far away on the billow—in determining the exact distance and direction of the part for which he is bound—whereas, the model of his perpetual motion is preserved in the museum as a lasting monument of this clock-maker's ignorance, perseverance, and handicraft.

It is another common error to suppose that genius alone can make a man a great mechanic, a great chemist, or a great any thing. Some one makes the remark, that every man is more than half humanity ; and we do believe that the differences of the degrees of know-

ledge of different men arise more from their difference of application, than from original differences of capacity. Let, therefore, the young workman earnestly try to learn, and we do assure him that he will make advances which will be proportional to his application.

This book has been written with the view of assisting the young workman in obtaining a knowledge of the calculations connected with machinery. The first part is directed to such parts of arithmetic as workmen generally require, and in which they are most commonly deficient. Nor is this to be wondered at, since the school books in our language contain, generally speaking, no explanation of the nature of the rules which they give, and are, moreover, embarrassed with so many divisions and subdivisions, that the mind of the scholar is perfectly perplexed, nor can it lay hold of the great leading principles which pervade the whole system. As this is the great instrument used throughout the book, we have endeavoured to make its use and management easily understood. The examples which we have given are indeed few and simple; but, if carefully considered, they will be found sufficient. The mere habit of calculation cannot be said to constitute a knowledge of arithmetic, it is easily obtained, but is of no avail without the principles. To construct a set of mathematical tables requires, not only a knowledge of principles, but also immense calculation. M. De Pronney was desired by the government of France, to construct a very large set of such tables; a task which would require the labour of a mathematician for many years. But Pronney fell upon an expedient which was every way worthy

of a man of science. A change in the fashions of the Parisians had thrown about five hundred wig-makers idle, and Pronney contrived at once to give employment to these barbers, and at the same time to serve the purposes of science. He digested the principles of the calculation of these tables into short and simple rules, and printed forms of them, which he gave into the hands of these workmen, who, in a few months, produced a set of tables, the most correct and extensive that ever has been made. Let not these statements induce you, however, to neglect the practice of calculation; on the contrary, improve yourself in it wherever you can, but be also careful to learn the principle.

In that part devoted to geometry, we have given such information without demonstration as was necessary to the right understanding of the rest of the book; and the like may be said of the conic sections, mensuration, and useful curves. Thus far the book may be said to be a compound of certain branches of the mathematics. But it is hoped that the reader, to whom such studies are new, will not be contented to stop here; but will be induced to study these subjects in theory; and for such as may be desirous of entering on a course of study, where there is nothing to be met with but unsophisticated truths connected together by the most beautiful relations, we intend to offer a few words of well-meant advice as to the order and means of prosecuting such studies.

In the first place, let the Elements of Euclid be studied so far as the end of the first book, in the course of which it should be borne in mind, that there is nothing

really difficult to be met with. The greatest difficulty is, we believe, this, that, to a proposition which is so simple as to be almost self-evident, there is often attached a long demonstration, which is apt to lead the reader to suppose that there is really something mysterious in it, which he does not understand. This proceeds from the fact, that it often requires a greater deal of circumlocution to show the connection of simple propositions with first principles, compared with propositions which are more complex ; but, we have no hesitation in saying, that if the steps of the propositions are carefully considered, one by one, they will be easily understood, and will lead at last to perfect conviction. This great length of the demonstrations of these propositions is not unnecessary, as some assert, but arises out of the nature of the thing; for, as Lord Brougham has well observed, in the preliminary discourse to the Library of Useful Knowledge, "Mathematical language is not only the simplest and most easily understood of any, but the shortest also." Of Euclid's Elements, there are various editions. Those of Simpson and Playfair are generally used in this country, and are deservedly popular. But we beg to recommend to the workman the edition of Mr Robert Wallace, of Glasgow, both for its execution and cheapness. The demonstrations are clear and short ; many new propositions are added, and the connection of theory with practice is never omitted where it can be introduced, and for these reasons we recommend it to the artizan.

When the first book of Euclid has been read, the study of algebra should be commenced, on which sub-

ject there are few good treatises to be found. That which we think best is the translation of Euler, a book which has come from the hand of a master, and is therefore characterized by great simplicity. Another good book, but long since published, and now not easily come at, is the treatise of Saunderson. Let either of these works, or others if they cannot be had, be read carefully so far as to equations of the second degree. When this has been carefully studied, the foundation will be laid of much valuable knowledge. If any one part of this department of algebra can be said to be difficult, it is that of powers and roots, which is a subject of the greatest importance; and should, on that account, receive the most careful attention; and, if the treatise of Euler be used, we have no hesitation in saying, little difficulty will be experienced. It may be necessary to observe, that attention should be paid all along to the intimate connection of arithmetic and algebra, which will tend to the better understanding of them both.

Having advanced thus far, Euclid must again be returned to; and, after revising the first book, read on to the sixth inclusive. Occasional revision of the algebra is recommended, and an advancement as far as equations of the third degree; after which Euclid may be read to the termination.

The study of trigonometry may then be introduced; on which subject we have various works of various merits. The treatise prefixed to Brown's Logarithmic Tables may be employed; and when it is understood, and the management of the logarithmic tables acquired, the works of Gregory, Lardner, or Thomson may be

consulted; the last is the most simple. After the study of trigonometry, Simpson's conic sections may be read with advantage.

Perhaps it may be a kind of relief at this stage, to see something of the application of mathematics to mechanics, and, for this purpose, the work of Keil on Physics, or the article, mechanics, in the second volume of Hutton's Mathematics. The neat little treatise of Mr Hay of Edinburgh will answer the same purpose exceedingly well.

But for the purpose of obtaining a good knowledge of theoretical mechanics, a more extensive knowledge of mathematics than we have hitherto supposed becomes absolutely necessary. A knowledge of the method of fluxions and fluents, or the differential and integral calculus, which bear a strong analogy to each other, and which have been employed for similar purposes. The simplest work on fluxions, and we believe the best, is the treatise of Simpson; and this may be followed by a perusal of Thomson's differential and integral calculus. And with this preparation the student may go on to read the first volume of Gregory's Mechanics, a book in which, we believe, he will find ample satisfaction. The second volume of this excellent work is almost entirely popular, and can cause no difficulty whatever. Another work, well worthy of a perusal, is that of Sir John Leslie, we allude to his Natural Philosophy; a work which, though neither strictly mathematical, nor strictly popular, yet contains much valuable information communicated in both ways. Indeed all the works of this great man, however much has been said against them

as to their style, will, nevertheless, be found to repay amply the trouble of a perusal.

We will not lengthen out these directions, as we conceive that when the student has advanced thus far he will be possessed of much valuable information, and will have a sufficient knowledge of both books and things to guide himself in his future inquiries. We say future inquiries, as it is our firm conviction that he who has advanced to the point we have considered, will be too deeply embued with a love of science, even for its own sake, ever to cease from its cultivation, so long as his mind is capable of cherishing one ray of its benign influence.

In the practical application of the mathematics to the useful arts, ample assistance will be derived from the work of Dupin, one volume of which has been translated from the French by Dr Birkbeck, the well-known founder of mechanics' institutions.

As the library of the workman cannot be very extensive, the few books which it contains should be well chosen. The treatises published by the Society for the Diffusion of Useful Knowledge cannot be too warmly recommended; and are easy of access from their cheapness and mode of publication. Indeed, the foundation of this society forms a most important era in the history of mankind; and we fondly hope, as we firmly believe, that the benevolent exertions of its talented members will be crowned with success.

The author of the following pages hopes that his work will be found useful to workmen in general; and though no book was ever written so as to meet the views of every man, yet he trusts that the artizan will

find much information in it which he daily requires, collected and compressed within a smaller compass than in any work of a similar nature. Should this book be deemed a failure, it must at least be acknowledged that its aim has been utility; and that to a class of men on whose intelligence, exertions, and welfare, the prosperity of the nation depends. There is, indeed, a strong competition between this and other kingdoms in the improvement of the arts and manufactures; and, although Britain still stands pre-eminent among the nations in this respect, yet she must not, on that account, relax her endeavours towards improvement, otherwise she will be seen left lagging behind. When we reflect on the circumstance, that it is to workmen themselves that we have ever been indebted for improvements in the arts, it is reasonable to expect that this is the source from whence such improvements will continue to flow; and among workmen it may be safely affirmed, that he who is the most intelligent will be the most likely to make improvements.

Add to these considerations, the fact, that there is a pleasure inseparable from the study of science, which is perfectly independent of all its other advantages, and that the poor man as well as the rich has a right to participate in its enjoyment. The diffusion of scientific knowledge among the working classes becomes thus not only a duty which every man owes to his country, but, besides this, it is an act of benevolence, as it tends to administer pleasure to a class of most useful men, who, in a multitude of cases, suffer grievous privations. The working man, however, should be made well aware, that

no exertion of any individual, or society of individuals, can be of any avail in the diffusion of knowledge, unless the working man shall make an earnest exertion. To the young mechanic we then say—earnestly endeavour to improve your mind; and enliven your spare hours by the cultivation of science; and should this little volume facilitate your progress in that manly employment, the desire of the author shall be fulfilled.

GLASGOW, 27th August, 1832.

THE

MECHANICS' CALCULATOR.

ARITHMETIC.

VULGAR FRACTIONS.

1. In many cases of division, after the quotient is obtained, there is a remainder, which is placed at the end of the quotient, above a small line with the divisor under it: thus—88 divided by 12 gives a quotient 7 and remainder 4, which is written 12) 88 ($7\frac{4}{12}$. Now, this $\frac{4}{12}$ is called a fraction; and it is placed in this way to show that 4 ought to be divided by 12. In all cases where we meet numbers written in this form, we conclude that the number above the line is to be divided by that under the line. This should be well borne in mind, as it is of the greatest use in obtaining a clear notion of fractions.

2. A fraction is said to express any number of the equal parts into which one whole is divided. It consists of two numbers—one placed above and the other below a small line. The upper number is called the Numerator, because it numerates how many parts the fraction expresses; and the under number is called the Denominator, because it expresses or denominates of what kind these parts are;—or, in other words, the denominator shows into how many parts one inch, foot, yard, mile—one whole any thing—is supposed to be divided; and the numerator shows how many of these parts are taken: as $\frac{4}{12}$ of a foot, for instance. The denominator shows that the foot is here divided into 12 equal

parts (inches); and the numerator 4, shows that four of these parts are taken—(4 inches).

3. If the numerator had been equal to the denominator, as $\frac{12}{12}$, then the value of the fraction would have been one whole (foot); and the numerator, being divided by the denominator, gives 1, as a quotient. In the fraction $\frac{14}{12}$ of a foot, the numerator is greater than the denominator, and the value of the fraction is greater than one: for the foot being divided into twelve equal parts (inches), and fourteen such parts (inches) being expressed by this fraction, its value is more than one foot; and the numerator being divided by the denominator, gives $1\frac{2}{12}$. Again, $\frac{6}{12}$ of a foot is just 6 inches, or one-half foot; and had the foot been divided into two equal parts, one of these parts would have been equal to $\frac{6}{12}$, or $\frac{1}{2}$ is equal to $\frac{6}{12}$. From this we may conclude, that when the numerator is equal to, less, or greater than the denominator, the value of the fraction is equal to, less, or greater than one whole. It is, then, not the numbers which express the numerator and denominator of a fraction, but the relation they bear to each other, that determines the real value of a fraction. $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{6}{12}$, are all equal, though expressed by different numbers,—the denominators of all the fractions being respectively doubles of their numerators.

4. From what has been said, it will easily be seen, that, if we multiply or divide both terms of any fraction by the same number, a new fraction will be found, equal to the first; thus—$\frac{4}{8}$, multiply both terms by 2, we get $\frac{8}{16}$, or divide them by 2, $\frac{2}{4}$, and these again by 2, $\frac{1}{2}$. All who know any thing of a common foot-rule will understand this, at sight.

5. The first use which we shall make of the principle last stated, is to bring two or more fractions to the same denominator, and that without altering their real values. For example, take $\frac{2}{3}$ and $\frac{3}{4}$ of a foot. Multiply both terms of the

first fraction $\frac{2}{3}$ by the denominator of the second, 4: we get $\frac{8}{12}$. Next multiply both terms of the second fraction by the denominator of the first fraction, that is $\frac{3}{4}$ by 3: the result is $\frac{9}{12}$. Now, it will be seen (from No. 4), that these two fractions, $\frac{8}{12}$ and $\frac{9}{12}$, are equal to the two $\frac{2}{3}$ and $\frac{3}{4}$,—with this additional advantage, however, that they have the same denominator, 12: the great use of which will be seen hereafter. A like process is employed in the case of three or more fractions: thus, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$,—multiply the terms of the first fraction by 4 and 5, the denominators of the second and third, we get $\frac{40}{60}$; next multiply the second $\frac{3}{4}$ by 3 and 5, the denominators of the first and third, we next get $\frac{45}{60}$; lastly, multiply the third by the denominators of the first and second, 3 and 4, we get $\frac{48}{60}$. It will be useful to look over what we have done.—In obtaining the numerators of the new fractions, we have multiplied each numerator in the former fractions by all the denominators except its own; and so also for the denominators. But 3 multiplied by 4, and 4 multiplied by 3, are the same thing, viz. 12: so, likewise, 3 multiplied by 4 multiplied by 5 is 60, and will be 60 in whatever order we take them—3 by 4 by 5, or 4 by 3 by 5, or 5 by 3 by 4; when, therefore, we have obtained one denominator, it is sufficient. Hence the usual rule to reduce fractions to a common denominator: Multiply each numerator by all the denominators except its own for new numerators, and all the denominators together for the common denominator.

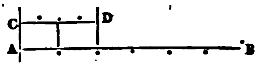
6. We are now prepared to add two or more fractions together. It is very easy to see how we may add $\frac{3}{8}$ and $\frac{4}{8}$ of an inch, and that their sum is $\frac{7}{8}$; but it is not quite so evident how we are to add $\frac{2}{3}$ and $\frac{3}{4}$ of a foot. If we had them, however, of one denomination, the difficulty would vanish. By No. 5, bring them to a common denominator—they stand thus: $\frac{8}{12}$ and $\frac{9}{12}$, or 8 and 9 inches; add the numerators, and under their sum place the denominator, $\frac{17}{12}$; divide the numerator by the denominator (No. 1), the quo-

tient is $1\frac{5}{12}$, or one foot five inches. The reason of bringing them to a common denominator is, that we cannot add unlike quantities together: and we do not add the denominators, their only use being to show of what kind the quantities are. The rule, then, is—bring the fractions to a common denominator, add the numerators together, and under their sum place the common denominator.

7. In subtraction we bring the fractions to a common denominator, and taking the lesser from the greater of the two numerators, place under their difference the common denominator. The reason of this may be easily inferred from (No. 6), $\frac{3}{8}$ subtracted from $\frac{1}{2}$, when brought to a common denominator, $\frac{6}{16}$ from $\frac{8}{16}$, the difference is $\frac{2}{16}$, equal to $\frac{1}{8}$, by No. 4.

8. To take one number as often as there are units in another, is to multiply the one number by the other. To multiply 4 by 2, is to take the number four two times, as there are two units in 2; and to multiply 4 by $\frac{1}{2}$, is to take four one-half times, or the half of four, as there is only half a unit in the fraction $\frac{1}{2}$. This may be thought so simple, that it needs not be stated; but let it be observed, that it explains a fact in the multiplication of fractions, which many excellent practical arithmeticians do not understand; viz. how that, when we multiply by a fraction, the product is less than the number multiplied. If the fraction $\frac{1}{2}$ is to be multiplied by $\frac{1}{4}$, (let the fractions both refer to an inch,) this is taking $\frac{1}{2}$ (inch) $\frac{1}{4}$ times, or taking the one-fourth part of one-half inch, which is one-eighth. The product $\frac{1}{8}$ is obtained by this simple process: multiply the numerators together for a new numerator, and the denominators together for a new denominator; the new fraction will be the product. That this is true in general may be shown by taking other fractions, thus: $\frac{2}{4}$ of $\frac{2}{6}$,—the product by the rule is $\frac{4}{24}$, which may be simplified by dividing the numerator and denominator by the same number, on the principle

of No. 4; if 4 be the divisor, the result is $\frac{1}{6}$, which is the same as $\frac{4}{24}$. Now, that $\frac{1}{6}$ is the real product of $\frac{2}{4}$ by $\frac{2}{6}$, may be shown thus: divide a line AB into six equal parts; take two of these parts, and join them by CD. Divide CD into four parts,



and it will be seen that the two parts of this line CD are just equal to one division on the line AB, or $\frac{2}{4}$ of CD is equal to $\frac{1}{6}$ of AB; so that $\frac{2}{4}$ of $\frac{2}{6}$ is $\frac{1}{6}$. The rule, then, is general.

9. Division is the reverse of multiplication; hence, to divide in fractions,—invert the divisor, and proceed as in multiplication. Thus, to divide $\frac{1}{2}$ by $\frac{1}{4}$, invert the divisor $\frac{1}{4}$, it becomes $\frac{4}{1}$, which multiplied by $\frac{1}{2}$ gives $\frac{1}{2}$ multiplied by $\frac{4}{1}$, equal to $\frac{4}{2}$; and by dividing, to make the fraction less, see No. 4,—$\frac{2}{1}$, which, by No. 1, is just 2 or twice. This is the quotient; and it is easily seen, if these fractions relate to a foot, that there are 2 quarters, or twice $\frac{1}{4}$ of a foot, in one-half foot, or $\frac{1}{2}$.

10. We have now endeavoured to explain the nature of the fundamental rules of Vulgar fractions, as simply as possible; but instances often occur, where it is necessary to prepare for these operations;—first, where whole numbers are concerned; and secondly, where the fractions are large, and, consequently, not so easily managed.

11. As to the first, where whole numbers are concerned, it is to be observed, that when unit, or 1, is used, either to multiply or divide a number, it does not change the value of that number. Thus, 6 multiplied by 1 is 6, and 6 divided by 1 is 6. According to the principle shown in No. 1, we may write the number 6 in this way, $\frac{6}{1}$, without altering its real value—with this advantage, that we have it now in the form of a fraction. We shall illustrate this by a few examples, and show that numbers, whether whole or fractions are in this department of arithmetic managed by the same rules.

Add 8 to $\frac{3}{4}$,—here we write them $\frac{8}{1}$ and $\frac{3}{4}$, which, brought to a common denominator, are $\frac{32}{4}$, $\frac{3}{4}$—their sum is $\frac{35}{4}$; then, by No. 1, divide the numerator by the denominator, we get $8\frac{3}{4}$, the number we set out from. $7\frac{1}{3}$, which is read seven and a third, may on the same principle be put in the form of a common fraction: for it is 7 wholes added to $\frac{1}{3}$ part of a whole, and may be thus written, $\frac{7}{1}$ and $\frac{1}{3}$, equal to $\frac{21}{3}$ and $\frac{1}{3}$, whose sum is $\frac{22}{3}$; divide the 22 by the 3, the result is $7\frac{1}{3}$, the first number. This very simple principle is often used, and deserves to be attended to; and whenever whole numbers occur, they must be managed after this manner, so that they may come in more easily in the course of calculation.

12. When the fractions are very large, it becomes necessary to bring them to a simple form, not only that we may more easily see their value, but that they may be more readily operated upon. Thus, $\frac{6}{72}$ is not so simple as $\frac{1}{12}$, nor so easily managed, and the one fraction is just equal in value to the other; for, by No. 4, the numerator and denominator of $\frac{6}{72}$ being both divided by 6, gives $\frac{1}{12}$. Also, $\frac{100}{2000}$, when 100 is used as a divisor, gives $\frac{1}{20}$. Whenever we can find a number which will divide both terms of the fraction without remainders, we ought to employ it, and thus make the fraction simpler in form, though of exactly the same value. The divisor thus used to simplify fractions, is usually called the common measure, and may sometimes be found at sight, although sometimes there is no such number at all. As in $\frac{2}{4}$, it is seen at once that 2 is the common measure; but in the fraction $\frac{3}{4}$ no such common measure can be found: consequently, the fraction cannot be made simpler. Sometimes, also, two or more numbers will divide the fraction; thus, $\frac{4}{8}$ may be divided by 4 or by 2—the greatest is preferred, because it brings the fraction to the lowest terms at once. When this cannot be attained at sight, the following rule may be employed: Divide the greater term by the less; if these leave any remainder, divide the

less term by it; and thus go on dividing the last divisor by the last remainder, and that divisor which leaves no remainder is the greatest common measure. This rule may be applied to the following example:

$\frac{1470}{2205}$ By the above rule.

$$\begin{array}{rrl}
1470\,) & 2205\,(\,1 & \\
 & \underline{1470} & \\
 & 735\,) & 1470\,(\,2 \\
 & & \underline{1470}
\end{array}$$

735 is the common measure; therefore,

$$735\,)\,\frac{1470}{2205}\,(\,\tfrac{2}{3},$$ the simple form of the fraction.

DECIMAL FRACTIONS.

13. Let us examine the number 3333 (three thousand, three hundred, thirty and three). The same figure is used, but for every place it is removed towards the left, its value is increased ten times; and, consequently, if we begin at the left hand side, and go on towards the right, we see that every figure has a value ten times less, than the same figure placed one place nearer the left,—each number expressing tenth parts of the number next it to the left. Hundreds are just tenth parts of thousands; tens are tenth parts of hundreds; and units are tenth parts of tens, &c. Now, the the same 3333, with a point placed before any of its figures, would still have the same property of each figure towards the right, having a tenth part of the value it should have had in the next place towards the left: that is, the point would have no effect in altering the relative value of the

figures; but it would have this effect, that the figure which stands at its right hand would signify units. Thus, 33·33, the same figures as before, with a point placed betwixt the middle two, and from what has been said, we conclude that the 3 to the left of the point is units. From this it follows, that the next 3 on the right of the point is tenth parts of ninety, and the 3 following that again tenth parts of a tenth part of ninety, or hundredth parts. Had it been written thus: 3·333, the last three to the right of the point would have been a tenth less again, &c.; so that all the figures that follow the point to the right are less than units, consequently, they are fractional; and from their decreasing by tenths each place, they are called Decimal fractions—from the Latin word *decem*, ten. Thus, then, $\frac{3}{10}$ may be written ·3.

14. It is to be observed here, that the use of the cypher (0) is in decimals quite similar to what it is in whole numbers,—where its only use is to remove some figure from the units' place, and therefore alter its value tenfold. Thus, in the number 40, the cypher of itself signifies nothing, but serves to remove the 4 to the tens' place. Had it been 04—here the cypher is of no use, because there is no figure to remove beyond it from units' place. The same is true of any number of units. Now, we have seen that ·3 is just $\frac{3}{10}$; and, from what has been said, it will follow, that ·03 is three hundredth parts, or $\frac{3}{100}$, as the cypher in ·03 removes the ·3 a place farther from the units' place towards the right and No. 13. makes it ten times less in value than it would have been one place nearer the left; or, it is now tenth parts of a tenth part. For the same reason ·003 is the same as $\frac{3}{1000}$.

15. The numbers 33 is read thirty and three, and ·33 is read three tenths and three hundredths, or sometimes thirty-three hundreds. Now, $\frac{3}{10}$ added to $\frac{3}{100}$ give, by No. 6, $\frac{330}{1000}$, which, simplified, is $\frac{33}{100}$ by No. 4. If we wished to write $\frac{3}{1000}$ in the other form, it is done simply thus: point 0

in tenth's place, 0 in hundredth's place, and 3 in thousandth's place; that is, ·003. Take, now, $\frac{4}{10}$ and $\frac{6}{100}$; adding, then, by No. 6, we get $\frac{460}{1000}$, simplified $\frac{46}{100}$, which, written with the point, is simply ·46. We may now see, that any number placed after the decimal point is a fraction; which may be expressed by a numerator which is that number, and a denominator consisting of 1, with as many cyphers annexed as there are figures in the numerator: so ·3034 is the same thing as $\frac{3034}{10000}$.

16. These simple statements being understood, all that follows will be easy. The principle being kept in mind, that the numbers to the right of the point have the same relation to one another,—every figure on the one side of the point as well as on the other, being ten times greater than it would have been in the next place to the right, and ten times less than in that to the left.

17. To add decimal fractions, we proceed just as in whole numbers, placing units under units, and consequently points under points, and carrying to each new column to the left, by 1 for every ten in the column already added. As $\frac{1}{2}$ may be written $\frac{5}{10}$ or ·5; $7\frac{1}{2}$ may, therefore, be written 7·5; $4\frac{1}{2}$ may be written 4·5. Now, add 7·5 and 4·5 by the rule we have given, and we will obtain a result which must be correct,—as may be proved by principles laid down in the former chapter. Here we have kept the points under each other and put a point in the answer just under the others, and the sum is 12, with no decimal fraction.

$$\begin{array}{r} 7{\cdot}5 \\ \underline{4{\cdot}5} \\ 12{\cdot}0 \end{array}$$

Take $7\frac{1}{2}$ and bring it to the form of a common vulgar fraction, by the principle No. 11, and it will be $\frac{15}{2}$; do so likewise with $4\frac{1}{2}$ and we get $\frac{9}{2}$; they have a common denominator, and add them by No. 6, we have $\frac{24}{2}$,—now, this fraction, by No. 4, is equal to $\frac{12}{1}$, or 12, the same as before. Take now 135·7, and 1·23, and ·764, and 9·102, and 8·003, and ·035; to find their sum. Here we place, as before, all the points under each

other, and proceed as in addition of whole numbers, carrying by tens and pointing the sum in the line under the other points:

135·7
1·23
·764
9·102
8·003
·035
154·834

18. Subtraction is managed in like manner as in common numbers, the same attention being paid to the points. Thus, subtract 33·785 from 1967·32; they are placed thus, and subtracted as in whole numbers, the point in the answer being placed in a line with the others. It is to be observed, that there are more decimal places in the under number than in the upper, and the deficiency may be supplied by adding cyphers to the upper line, which, as there is no significant figure beyond, does not alter the value of the number.

1967·320
33·785
1933·535

19. Multiplication of decimal fractions is performed as in whole numbers, paying no attention to the points until the product is obtained, when we point off as many places from the right hand side of the product, as there are decimal places in both the multiplicand and the number which multiplies, or multiplier. Thus, multiply 36·42 by 4·7. Here ·174 are pointed off as decimals, as there are two decimal places in the multiplicand and one in the multiplier—in all three. That this rule is correct, may be inferred from the results of a former example in No. 8. Here we multiplied 4 by $\frac{1}{2}$, and found the product to be 2: now, $\frac{1}{2}$ is equal to $\frac{5}{10}$, which may be written ·5;

36·42
4·7
25494
14568
171·174

then let us multiply 4 by ·5, as directed above, and we will find the same result, 2; where, by principle 14, the cypher being pointed off, there remains 2—a whole number.	4 ·5 2·0

20. Division may be properly defined, the finding of one number (the quotient), such, that when multiplied by another (the divisor), will give a product equal to a third (the dividend). The dividend may thus be viewed as the product of the quotient and divisor; hence, the quotient and divisor should, together, contain as many decimal places as the dividend. This being observed, the rule will be easily followed: Divide as in whole numbers, and when the quotient is obtained, point off from the right as many places for decimals as those of the divisor want of those in the dividend. Divide 22·578 by 48·6, the quotient $4·6\frac{22·2}{48·6}$ is obtained by common division, and pointed

$$48·6)22·578(4·6\frac{22·2}{48·6}$$

thus, because the divisor wants only one decimal place to have as many as the dividend. In many cases, when the quotient is obtained, there will not be as many figures as make up the number of decimal places required; here we must place one or more cyphers, betwixt the point and the quotient figures, so as to make up the number required. Thus, divide 1·0384 by 236, the quotient is 44—only two places, whereas there should be four decimals in the quotient; because there are four in the dividend and none in the divisor. We, therefore, place the quotient thus,—·0044; and to prove that this is the true quotient, we have only to multiply it by the divisor, and the product being the same as the dividend, the operation must be correct.

21. From the great facility with which decimal fractions may be managed, it is very desirable that we could bring vulgar fractions to the same form, in order that they might more easily be wrought with. Now, this may be done on

the principles already laid down:—take the fraction $\frac{1}{8}$, and, on the principle No. 4, multiply both terms by 1000, it then becames $\frac{1000}{8000}$, which is equal to $\frac{1}{8}$; divide (No. 4.) both numerator and denominator by 8; then $8)\frac{1000}{8000}(\frac{125}{1000}$, which last fraction is expressed in the decimal notation thus, (on the principle of No. 15,) ·125, which, from the way it has been derived, must be equal to $\frac{1}{8}$. This may, however, be found more immediately thus: add as many cyphers to the numerator as you find necessary, and divide by the denominator thus,—8)1000(·125. If we have only to add one cypher, before we get a quotient figure, we put a point in the quotient; but if more, then we put as many cyphers in the quotient after the point. Thus, $\frac{1}{25}$—25)100(·04, and $\frac{1}{25}$ is just $\frac{4}{100}$, or ·04.

22. In many cases the quotient would go on without end; but it is to be observed, that it is not necessary to continue any operation in decimals, at least in mechanical calculations, beyond three or four places, as ten thousandth parts are seldom necessary to be considered in practice. For similar reasons, it is unnecessary to give rules for repeating and circulating decimals: carry them to four places, and it is all that is necessary.

Other applications of these principles will be found in the next chapter, on Compound numbers.

COMPOUND NUMBERS.

23. In mechanical calculations, we are often concerned with weights and measures, and it is necessary to know how to operate with the numbers which express these. The rules given in books of arithmetic are generally very long, and therefore, not very easily understood; yet the steps of the

operation are simple. We shall therefore show the mode of procedure, in some very easy examples, and the reader will find no difficulty in applying the principles he may thus imbibe to more complex cases.

24. If we have to add 9 yards 2 feet 6 inches, to 2 yards 1 foot 3 inches, 8 yards 0 feet 11 inches long measure. Then we must in this, as in all other cases of compound addition, arrange them in order,

yds.	feet	inch.
9	2	6
2	1	3
8	0	11
20	1	8

the greater towards the left hand, and the lesser towards the right, and there must be a column for every denomination of weight or measure, in which column the respective quantities must stand, so that feet will stand under feet, inches under inches; pounds under pounds, and ounces under ounces, &c. Add now the column toward the right, which in this example amounts to 20 inches, or 1 foot 8 inches, we therefore put down the 8 inches under the column of inches, and add the 1 foot to the column of feet, which comes to 4 feet; that is 1 yard and 1 foot. The 1 foot is put down under the column of feet, and the 1 yard is added, or carried as it is usually called, to the column of yards, whose sum is 20.

If we have to add 2 tons 2 cwt. 1 quar. 17 lbs. 11 oz. avoirdupois, to 12 ton 10 cwt. 2 lbs. 2 oz., 2 cwt. 1 quar. 18 lbs. 3 oz.,

tons	cwt.	quar.	lbs.	oz.
2	2	1	17	10
12	10	0	2	2
0	2	1	18	3
0	0	0	9	11
14	14	3	19	10

and 9 lbs. 11 oz.; then, from what was remarked above, they will be put down as in the margin. Then the sum of the right hand column is 26 oz., which is 1 lb. 10 oz., we put down the 10 in the column of oz., and carry the 1 lb. to the column of lbs. which is next; and this when added comes to 47 lbs., that is 1 quar. and 19 lbs.; the 19 is put in the column of lbs. and the 1 is carried to that of quars., which comes to 3, which not amounting to 1 cwt. we put

down the 3 in the column of quars. and carry nothing to the column of cwts., which when added, amounts to 14, this we put down, and as it does not amount to 20 cwt. or 1 ton, we carry nothing to the column of tons; and when this column is added, its sum is 14.

25. In Subtraction the same principle of arrangement is to be observed, and the lesser quantity is to be put under the greater. If we have to subtract 1 ton 13 cw. 2 quars. 17 lbs. 12 oz., from 9 tons 8 cwt. 1 quar. 4 lbs. 7 oz. avoirdupois. They are arranged as in the margin. We begin to subtract at the lowest denomination, viz. oz.—12 oz. from 7 oz. we cannot, but we add a lb. or 16 oz. to the 7, which is supposed to be borrowed from the column of lbs. which stands next it, towards the left; now 16 added to 7 makes 23, and 12 from 23 leaves 11, which is put down in the column of oz. Now we must pay back to the column of lbs. the pound or 16 oz. which we borrowed, therefore it is 18 from 4. Here we have to borrow from the column of quars., and 1 quar. being 28 lbs. we borrow 28, then 28 and 4 are 32, therefore 18 from 32 leaves 14, which is put down, and the 1 quar. paid back to the column of quars.; 3 from 1, we must borrow 1 cwt. or 4 quars., therefore 3 from 5 and 2 remains, which is put down. Add 1 to 13 for the 1 cwt. that was borrowed, then 14 from 8, we cannot, but borrow 20 from the next column, then 14 from 28 and 14 remains. Pay back to the column of tons the 1 ton, or 20 cwt. which we borrowed, then 2 from 9 and 7 remains, which is put down.

tons	cwt.	quars.	lbs.	oz.
9	8	1	4	7
1	13	2	17	12
7	14	2	14	11

The same principle holds in other examples, the only variation being that the numbers to be borrowed from the next higher column, will depend upon the relative values of these columns, which may be known by examining a table of the particular weight or measure, to which the example may refer.

26. In Multiplication, which is only a short way of performing addition in particular cases; the principles are nearly similar; thus, to multiply 3 tons 2 cwt. 2 quars. 6 lbs. 10 oz. by 3; they are arranged as in margin. Then the first product is 30 oz. or 1 lb. which is carried to the column of lbs., and 14 oz. which is put down in the column of oz. The product of the lbs. is 18, and the one lb. carried is 19, which not amounting to 28 lbs. or 1 quar., nothing is to be carried to the column of quars. The product of the quars. is 6, which is 1 cwt. to be carried and 2 quars. to be put down. The product of cwts. is 6, and the one carried from the former column makes 7, nothing being carried; the column of tons is 9. By examining the following examples, and referring to the tables of weights and measures, the general application may be easily inferred.

tons	cwt.	quars.	lbs.	oz.
3	2	2	6	10
				3
9	7	2	19	14

	Degrees.	min.	seconds.	yds.	feet.	inch.	8th parts.
	23	14	17	17	2	9	6
			6				5
	139	25	42	89	2	0	6
Carry by		60	60		3	12	8

27. It may not be out of place, here, to notice, Duodecimal, or what is commonly called, cross multiplication; which is very useful to artificers in general, in measuring timber, &c. The foot is divided into 12 inches, each inch into 12 parts, and each part again into 12 seconds; these last, however, are so small, that they are generally neglected in calculation.

If we wish to find the surface of a plank, whose breadth is 1 foot 7 inches, and length 8 feet 5 inches, we place the one under the other, feet under feet, inches under inches, &c. as in the margin. Multiply the inches and feet, in the upper line, by the feet

8	5	
1	7	
8	5	
4	10	11
13	3	11

in the under line, placing the product of the inches, under the inches, and that of the feet, under the feet. Then multiply the inches and feet, of the upper line, by the inches in the under line, placing the product one place further towards the right, and carry by 12ves where necessary; as in this example, 7 times 5 is 35, that is, two 12ves and 11 over; the 11 is put down, and the 2 added to the product of the next column,—7 times 8 is 56, and the 2 carried makes 58, that is four 12ves and 10 over; the 10 is put down, and the 4 carried to the next column. These are now added, observing again to carry by 12ves.

feet.	inch.	
4	7	
8	4	
36	8	
1	6	4
38	2	4

feet.	inch.	parts.		
35	4	6		
12	3	4		
424	6	0		
8	10	1	6	
	11	9	6	0
434	3	11	0	0

The feet in the example are square feet, but the inches are not square, as might be thought at first sight, but 12th parts of a square foot; and also the numbers standing in the third place, are 12th parts of these 12 parts of a foot, and so on.

28. Before we consider the Division of compound numbers, it will be necessary to attend a little to the nature of reduction. This is usually thought by beginners to be very perplexing, but a little attention to the principle, will obviate all this apparent difficulty.

In every lineal foot there are 12 inches, and therefore there will be 12 times as many inches, in any number of feet, as there are feet; thus, in 8 feet there are 8 times 12 inches, that is 96 inches. In every lb. avoirdupois, there are 16 ounces, therefore in 18 lbs. there are 18 times 16

ounces, that is 288 ounces. So that we multiply the higher denomination, by the number of the lower that make one of the higher, and the product is the number of the lower contained in the number of the higher, which we multiply. In the previous examples, feet and pounds are the higher denominations, and inches and ounces are the lower. From these remarks it will be easy to see, how we proceed in finding the number of 8 parts of an inch contained in 3 yards 2 feet 7 inches and $\frac{5}{8}$ parts, long measure. Bring the yards to feet, 3 multiplied by 3 are 9, to which we add the 2 feet, which make 11. This brought to inches, is 11 multiplied by 12 or 132, to which we add the 7 inches, making 139. This brought to 8 parts gives 139, multiplied by 8, that is 1112, to which we add the 5 eight parts, making 1117 the answer.

The examples subjoined are managed in a like manner. The multipliers varying with the kind of weight or measure.

cwt.	quar.	lbs.
27	1	22
4 mult.		
108 quars.		
1 add		
109 quars.		
28 mult.		
3052 lbs.		
22 add		
3074 lbs.		

acres.	roods.	poles.
22	3	24
4 mult.		
88 roods		
3 add		
91 roods		
40 mult.		
3640 poles		
24 add		
3664 poles.		

The work is reversed, when we wish to ascertain how many of a higher denomination are contained in any number of a lower. Thus, in 1440 inches, long measure, there will be one foot for every 12 inches, we therefore divide 1440 by 12, and the quotient will be the number of feet,

that is 120 feet. Then there is no remainder, but if there had, it would have been of the same kind with the dividend, that is inches. In the same way find how many tons, cwts., quars., and lbs., are contained in 12345678 oz.

oz. in 1 lb.—16	)12345678	ounces.
lbs. in 1 quar.—28	)771604 lbs.—14 oz.	
quars. in 1 cwt.—4	)27557 quars.—8 lbs.	
cwt. in 1 ton—20	)6889 cwt.—1 quar.	
	344 tons—9 cwt.	

The answer therefore is 344 tons 9 cwt. 1 quar. 8 lb. 14 oz.—which may be proved by reducing the work to ounces by the method given above.

29. It is frequently of great use, to express compound numbers fractionally; thus, so many feet and inches as the fraction of a yard. What fraction of a yard is 2 feet 8 inches? Now, from what has been said on vulgar fractions, it will be easily seen that one yard is here the unit, or denominator of the fraction, which must of course be brought to inches. Now there are 36 inches in one yard, which must be the denominator of the fraction, and the numerator will be the quantity taken; that is 2 feet 8 inches reduced to inches, or 32 inches. The fraction therefore is $\frac{32}{36}$, or simplified $\frac{8}{9}$, which, turned into a decimal, is 0·8888, one yard being 1. So likewise, what fraction of a cwt. is 2 quars. 14 lbs. 3 oz.? This last reduced to ounces is 1123, which is the numerator of the fraction, and the denominator is 1 cwt. reduced to oz., or 1792 oz.; the fraction is therefore $\frac{1123}{1792}$, which is expressed decimally 0·6264. We think that these examples will be sufficient to shew the mode of procedure, and it remains for us to consider the reverse of this; to estimate the value of such fractions in terms of the weight or measure to which they refer.

30. It will be easily seen, that one half of a foot is twelve

times greater than one half of an inch, or that any given part of a foot, is a twelve times greater part of an inch; thus, $\frac{1}{2}$ of a foot is $\frac{12}{2}$ of an inch; so that to bring any fraction of a foot to the fraction of an inch, we have only to multiply the numerator by 12. So likewise $\frac{1}{4}$ of a pound avoirdupois, is $\frac{16}{4}$ of an ounce, and $\frac{1}{5}$ of a yard is $\frac{3}{5}$ of a foot, or $\frac{36}{5}$ of an inch; and if we divide the numerator by the denominator, we get in the last example $\frac{36}{5}$ of a yard, equivalent to $7\frac{1}{5}$ inches.

What is the value of $\frac{1}{3}$ of 1 cwt.? By applying the foregoing principle it will be found that $\frac{1}{3}$ of 1 cwt. is $\frac{4}{3}$ of a quar., or a 28 times greater part of 1 lb., that is $\frac{112}{3}$; that is $37\frac{1}{3}$ lbs.—also $\frac{2}{3}$ of 1 lb. is 16 times $\frac{2}{3}$ of an ounce, or $\frac{32}{3}$ equal to $10\frac{2}{3}$ ounces.

31. It will generally be found best to express these decimally, thus the last example will be $\frac{1}{3}$ of a cwt., or 0.333 of a cwt., or 1·333 of a quar., or 37·666 of a pound. Thus it appears that any fraction of a cwt. is 4 times greater than a like fraction of a quarter, and any fraction of a quarter is 28 times greater than a similar fraction of a pound; hence, to reduce a fraction of a higher to its value in a lower denomination, we multiply the numerator of the fraction, by that number which expresses how many of the lower are contained in one of the higher, while the denominator remains unaltered. On the other hand, to bring a fraction from a lower to a higher denomination, the numerator remains the same; but we multiply the denominator, by that number, which expresses how many of the lower is contained in one of the higher. Thus $\frac{1}{3}$ of an inch is $\frac{1}{36}$ of a foot, or $\frac{1}{108}$ of a yard; or expressed in decimals 0·3333 of an inch, or 0·0277 of a foot, or 0·00924 of a yard.

32. On a like principle the value of a decimal expressing weight or measure, may be determined, simply by multiplying the decimal, by that number of the next twice, which is contained in one of the higher, and cutting off the proper number of decimals in the product,—thus:

```
 ·37689 of a cwt.
      4
 -------
 1·50756 quarters.
      28
 -------
14·21168 pounds.
      16
 -------
 3·38688
```

Here it will be observed, that the whole numbers cut off are not multiplied, and the value of ·37689 of a cwt. is 1 quar. 14 lbs. 3·386 oz.

We will conclude this chapter on compound numbers, by some remarks on Division. The same arrangement is to be observed here as in addition. The greater quantity being towards the left of the lesser.

Let it be required to divide 13 yards 2 feet 8 inches by 4. We say 4 in 13, 3 times and 1 over, that is one yard, which must be reduced to feet, the next lower denomination; that is 3 feet, and the 2 feet are five feet—now 4 in 5, 1 and 1 over, which last being a foot, must be reduced to inches, it is therefore 12 inches, and the 8 make 20: then 4 in 20, 5 times; the answer therefore is 3 yards, 1 foot, 5 inches.

```
    yards.  feet.  inch.   yards.  feet.  inch.
3)   16      2      9  (     5      1      11
     15
    ---
      1
      3 mult.
    ---
      3
      2 add
    ---
3)    5
      3
    ---
      2
     12 mult.
    ---
     24
      9 add
    ---
3)   33
     33
```

POWERS AND ROOTS.

32. The square of any number is the product of that number multiplied by itself: thus, the square of 2 is 4, the square of 4 is 16, the square of 5 is 25, &c. The cube of any number is the product of that number multiplied twice by itself: thus, the cube of 2 is 16, the cube of 3 is 27, the cube of 4 is 64, &c. On the other hand, when we talk of the square and cube roots of any numbers, we mean such numbers that, when squared or cubed, will produce these numbers; thus, 2 is the square root of 4, 3 is the square root of 9, and 4 is the square root of 16, &c. In like manner, 3 is the cube root of 27, 4 the cube root of 64, 5 the cube root of 115, &c. The cube and cube root are said to be of higher order than the square and square root; and there are higher orders than these, with which we shall not concern ourselves, as they will not occur in our calculations.—The method of raising any number to the square and cube powers, will be sufficiently obvious, from what has been said above; but the method of extracting the square and cube roots is not by any means so easy. We shall give the rules for the extraction of these roots; and as they are long, we would recommend the beginner to compare carefully each step in the examples, with that part of the rule to which it refers; and by doing so attentively, he will find that the greater part of the difficulty will vanish.

33. The rule for extracting the square root is this;

First—Commencing at the unit figure, point off periods of two figures each, till all the figures in the given numbers are exhausted. The second point will be above hundreds in whole numbers, and hundreds in decimals.

Second—If the first period towards the left be a complete square, then put its square root at the end of the given num-

ber, by way of quotient; and if the first period is not a complete square, take the square root of the next less square.

Third—Square this root now found, and subtract the square from the first period; to the remainder annex the next period for a dividend, and for part of a divisor double the root already obtained.

Fourth—Try how often this part of the divisor now found is contained in the dividend, omitting the last figure, and annex the quotient thus found, not only to the root last found, but also to the divisor last used.

Fifth—Then multiply and subtract, as in division; to the remainder bring down the next period, and, adding to the divisor the figure of the root last found, proceed as before.

Sixth—Continue this process till all the figures in the given number have been used; and if any thing remain, proceed in the same manner to find decimals—adding two cyphers to find each figure.

The square root of 365 is required.

	365 (19·1049
	1
29	265
9	261
381	400
1	381
38204	190000
4	152816
382089	3718400
9	3438801
382098	279599

The square root of 2 to six decimals is

	2(1·414213
	1
24	100
4	96
281	400
1	281
2824	11900
4	11296
28282	60400
2	56564
282841	383600
1	282841
2828423	100759

34. The easiest rule for the extraction of the cube root, is this:

By trials, take the nearest cube to the given number, whether it be greater or less, and call it the assumed cube. Thus, if 29 was the given cube whose root was to be extracted, then, 3 times 3 times 3, or 27, is the nearest less cube, and 4 times 4 times 4, or 64, is the nearest greatest cube; 27 is the nearer of the two, therefore, 27 is the assumed cube.

Add double the given cube to the assumed cube, and multiply this sum by the root of the assumed cube, and this product divided by the given cube, added *to* twice the assumed cube, will give a quotient which will be the required root, nearly.

By using, in like manner, the cube of the last answer, as an assumed root, and proceeding in the same manner, we will get a second answer nearer the truth than the first, and so on.

Find the cube root of 21035·8.

If 20 is assumed, its cube is 8000; if 30, its cube is 27000;—the one a great deal too small and the other too great: let us

therefore try some number between them, as 27; the cube of this is 19683, which we shall call the assumed cube, then.

Twice the assumed cube is 39366. Twice the given cube is 42071·6.

Therefore, the sum of the given cube and twice the assumed cube is 60401·8, and the sum of the assumed cube and twice the given cube is 61754·6.

Wherefore, by the rule,

```
             61754·6
                  27
             -------
             4322822
            1235902
            -------
60401·8 ) 1667374·2 ( 27·6047
```

This quotient is the root nearly; and by using 27·6047 in the same way that we used 27, we will get an answer still nearer the true root.

THE SLIDING RULE.

35. We are indebted for the invention of this useful instrument to Edmond Gunter. It is a kind of logarithmic table, whose great use is to obtain the solution of arithmetical questions by inspection, in the multiplication, division, and extraction of the roots of numbers. It consists of two equal pieces of boxwood, each 12 inches long, joined together by a brass folding joint. In one of those pieces there is a brass slider. On the face of this instrument, there are engraven four lines, marked by the letters A, B, C, and D; at the beginning of each line, the lines A and D being marked on the wood part of the rule, and B and C on the brass slider.

36. Before the use of the sliding rule can be explained, it is necessary that a correct idea should be formed of the method of estimating the values of the several divisions on

these lines. Let it be observed, then, that whatever value is given to the first 1 from the left, the numbers following, viz. 2, 3, 4, 5, &c., will represent twice, thrice, four times, &c., that value. If 1 is reckoned 1 or unit, then 2, 3, 4, &c., will just signify two, three, four, &c.; but if 1 is reckoned 10, then 2, 3, 4, &c., will represent 20, 30, 40, &c. If the first 1 is reckoned 100, then 2, 3, 4, &c., will represent 200, 300, 400, &c. The value of the 1 in the middle of the line is always ten times that of the first 1; the value of the second 2 is ten times that of the first 2: so that if the value of the first 1 be 10, that of the second 1 will be 100; the first 2 will be 20, and the second 2 will be 200, &c. The value of these divisions being understood, we may now attend to the minute divisions between these. Now, on the lines A, B, and C, there are 50 small divisions betwixt 1 and 2, 2 and 3, 3 and 4, &c.; and it follows, from the nature of the larger divisions, that if the first 1 be reckoned 1, or unit, each of these small divisions between 1 and 2, 2 and 3, &c., will be $\frac{1}{50}$, or ·02; and supposing still the first 1 to be unity, then the small divisions from the second 1 to 2, 2 to 3, &c., will each be ten times greater than a $\frac{1}{50}$, or ·02, that is, each of them will be $\frac{10}{50}$, or $\frac{1}{5}$, or ·2. In the same way, if the first 1 represents 100, the first 2 will be 200; if the second 1 will be 1000, the second 2 will be 2000, &c.; and on the same principle as above the small divisions or 50th part will represent each $\frac{1}{50}$ of 100, or 2, in the first half, or from the first 1 to 2, 2 to 3, &c., and $\frac{1}{50}$ of 1000, or 20, in the second half; or from the second 1 to the second 2, 2 to 3, &c.

37. These divisions being understood, we may proceed to show the method of using this rule in the solution of arithmetical questions.

38. To find the product of two numbers.

Move the slider, so that 1 on B stands against one of the factors on A; then the product will be found on the line A, against the other factor on the line B.

Thus, to find the product of 3 by 8:

Set 1 on B to 3 on A; then against 8 on B will be found the product 24 on A.

For the product of 34 by 16:

Set 1 on B against 16 on A, then look on B for 34, and against it on the line A will be found the product 544.

39. To find the quotient of two numbers.

This may be done in two ways,—either set 1 on the slider B against the divisor on A, then against the dividend on A the quotient will be found on B. Or, set the divisor on B against 1 on A, then the quotient will be found on A against the dividend on B; therefore, in general, it is to be remembered, that the quotient must always be found on the same line on which 1 was taken, and the divisor and dividend on the other line.

Thus, to find the quotient of 96 divided by 6:

Move the slider till 1 on B stands against 6 on A; then the quotient 16 will be found on B against the dividend 96 on A.

In like manner, to find the quotient of 108 divided by 12, we may take the latter form of the rule, thus:

Set 12 on B against 1 on A; then on the line A will be found the quotient 9 against 96 on B.

40. To solve questions in the rule of three or simple proportion:

Set the first term on the slider B to the second on A; then on the line A will be found the fourth term, standing against the third term on B.

If 4 lbs. of brass cost 36 pence, what will 12 lbs. cost?

Move the slider so, that 4 on B will stand against 12 on A; then against 36 on B will be found the fourth term 108 on A.

41. To extract the square root:

Move the slider so, that the middle division on C, which is

marked 1 stands against 10 on the line D, then against the given number on C the square root will be found on D.

It is to be observed before applying this rule, that if the given number consists of an even number of places of figures, as two, four, six, &c., it is to be found on the left hand part of the line C; but if it consists of any odd number of places, as three, five, seven, &c., it is to be found on the right hand side of C, 1 being the middle point of the line.

To find the square root of 81:

Here the number of places are even, being two; therefore, the number 81 is sought for on the left hand side of the line C.

Set 1 on C against 10 on D; then against 81 on C will be found 9 the square root on D.

For the square root of 144:

Set 1 on C to 10 on D; then against 144 on C will be found the square root 12 on D.

42. To find the area of a board or plank:

The rule is, to multiply the length by the breadth, the product will be the area; therefore, by the sliding rule,

Set 12 on B against the breadth in inches on A; then on the line A will be found the surface in square feet, against the length in feet on the line B.

To find the area of a plank 18 inches broad and 10 feet 3 inches long:

Move the slider so, that 12 on B stands against 18 on A; then will 10¼ on B stand against 15¾ on A, which 15¾ is square feet.

This may be proved by cross multiplication.

10	3	
1	6	
10	3	
5	1	6
15	4	6

43. For the solid content of timber.

The rule is to multiply length, breadth, and thickness all together.

Set the length in feet on C to 12 on D; then on C will be found the content in feet against the square root of the product of the depth and breadth in inches on D.

What is the content of a square log of timber, the length of which is ten feet, and the side of its square base is 15 inches.

Set 10 on C against 12 on D; then will 15 on D stand against the content $15\frac{5}{8}$ on C.

44. Other particulars on the measurement of timber will be given hereafter, when we come to mensuration.

MARKS OF CONTRACTION.

45. We earnestly request that particular attention be paid to this, not because it is difficult, but because it is of the greatest importance to the clear understanding of what follows in this book, and contributes greatly towards its shortness and simplicity.

46. When we mean to say that one thing is equal to another, we use this mark, $=$ thus, 3 added to $5 = 8$, is read thus, 3 added to 5 is equal to 8.

47. But the words, added to, may also be represented by a mark, $+$ thus $3 + 5 = 8$, is read, 3 added to 5 is equal to 8.

48. So likewise the difference of two numbers may be represented by a mark, —, which is a short way of expressing the word subtract, thus, $5 - 3 = 2$, is read, 5 subtract 3 the difference is equal 2; and thus $3+6-2=7$ is a short way of writing to 3 add 6 and subtract 2, the result is equal to 7.

49. After the same manner the mark $\times$, is used instead

of the words multiply by, thus, 3×2 is read 3 multiplied by 2.

50. To show that the operation of division is to be performed this mark is sometimes used, viz. $\div$, which is a short way of writing the words, divide by, thus, $15\div3=5$ is read, 15 divided by 3 is equal to 5: but we will in general place the divisor below a line with the dividend above it, on the principle stated in vulgar fractions, thus $\frac{15}{3}=5$ the same as $15\div3=5$.

51. The square of any number or quantity is marked by a small ^{2} placed at its upper right hand corner, thus, $3^2=9$ is read, the square of 3 is 9, also the cube is marked by a 3 in the same way, as $3^3=27$, that is, the cube of 3 is 27.

The square root is noted in a similar manner by the fraction $^{\frac{1}{2}}$ placed in the same way, as $9^{\frac{1}{2}}=3$, and so likewise the cube root, as $27^{\frac{1}{3}}=3$.

52. Parentheses, ( ) are used to show that all the numbers within them are to be operated upon as if they were only one; thus, $3+2\times5$, means that 3 is to be added to the product of 2 and 5, that is the amount of this is 13; but $(3+2)\times5$, means that 3 and 2 that is 5, is to be multiplied by 5, and the result will be 25; a very different thing from what it was before, which arises entirely from the use of parentheses. In like manner $3+2^2=7$, but $(3+2)^2=25$; here, as in every other case, the whole of the numbers within the parentheses are taken as one whole, and as such are affected by whatever is without the parentheses.

53. The rule for the measurement of the surface of timber given in our remarks on the sliding rule, may be expressed thus, length $\times$ breadth $=$ area; and the rule for simple proportion, to be given in the next chapter, may also be written thus:

$$\frac{\text{Second term} \times \text{third term,}}{\text{first term,}} = \text{fourth term.}$$

54. It is obvious that this is merely a kind of short hand

which might be carried still farther; for instance, in the last example we might make F stand for the first term, S for the second, T for the third, and £ for the last, and the rule would then be.

$$\frac{S \times T}{F} = £$$

55. We again insist that the young reader will read this chapter carefully over.

PROPORTION.

56. When four numbers following each other are such that the first is as many times greater or less then the second, as the third is greater or less than the fourth, they are said to be in proportion; thus, 2, 4, 3, 6, usually written thus, 2 : 4 :: 3 : 6; the mark : being put for the words, is to, and :: for, as, so that this would be read, 2 is to 4 as 3 is to 6. Here the first is half the second, and the third is half the fourth, and they are therefore in proportion; but they may be arranged otherwise and yet be in proportion, thus, 4 : 2 :: 6 : 3, where the first is twice as large as the second, and the third is twice as large as the fourth. In all cases the two middle terms are called the means, and the two outer terms are called the extremes. The product of the two means is equal to that of the two extremes, thus in the last example, $2 \times 6 = 12$ and $4 \times 3 = 12$. Now if we wanted the last term, to wit 3, it could easily be found by means of this property of numbers in proportion. If we had only three terms given, as 4 : 2 :: 6 : to find the fourth in proportion, which is the last extreme, and 4 is the first extreme. Now we must find such a number, that, when multiplied by 4, the product will be equal to the product of the means; $2 \times 6 = 12$, to find such a number we have only, by the

definition of division, to divide the product of the two means, viz. 12 by the first extreme 4, and the quotient 3 will be the answer. So universally $6 : 9 :: 12 :$ where the last term will be found, as before, by multiplying the two means $12 \times 9 = 108$, and dividing the product 108 by the first extreme 6, the quotient will be the last extreme 18, hence $6 : 9 :: 12 : 18$. The rule may be expressed simply thus; let F stand for the first term, S the second, T the third and £ the last, then we have $\frac{S \times T}{F} = £$, and this rule holds true whether the numbers be whole or fractional; and here it may be observed, that it will in most if not in all cases be best, to turn all vulgar fractions, when they occur, into decimals; thus, $2\frac{1}{2} : 3\frac{2}{3} :: 6\frac{1}{4} :$ or $\frac{5}{2} : \frac{11}{3} :: \frac{25}{4} :$

$$\left.\begin{array}{l} 2\frac{1}{2} = \frac{5}{2} = 2{\cdot}5 \\ 3\frac{2}{3} = \frac{11}{3} = 3{\cdot}666 \\ 6\frac{1}{4} = \frac{25}{4} = 6{\cdot}25 \end{array}\right\} \; 2{\cdot}5 : 3{\cdot}666 :: 6{\cdot}25 :$$

Here the mode of determining the fourth term is the same in all; the two means being $\times$, and their product $\div$, by the first term. This is usually called the rule of three, and is of the utmost utility in practical arithmetic. We shall now show how it is to be applied.

If we pay 40 pence for 2 feet of wood, how much will we pay for 6 feet at the same rate? Here it is clear we will pay in proportion to the quantity of wood; for as many times as we have 2 feet, we will pay so many times 40 pence; that is, the price will be in proportion to the quantity of wood. So that we may say, as the one quantity of wood is to another quantity, so will be the price of the first quantity to the price of the second. Hence the terms in the question will stand arranged thus:—$2 : 6 :: 40 : 120$, which term 120 is the price of 6 feet, and is found by the rule given above,

$$\text{thus, } \frac{6 \times 40}{2} = 120$$

57. In every question in simple proportion, there will always be three terms, one of which is of the same kind with the answer sought, whether it be money, measure, time, force, or any thing, which term in the question we place in the third place; as in the last question the answer was to be money, and therefore the money in the question, 40 pence, was placed in the third place. When this is done, we next consider whether the answer will be greater or less than this, and place the greater or less of the other two terms next it in the second place, and the other one first as the answer may require; after which, employ the rule given above, to find the answer.

58. As, for example, 40 men will do a piece of work in 15 days, in how many days will 20 men do the same? Here the answer must be days; consequently, 15 goes in the third term, and 20 men will take more time than 40 to do it, therefore we must put the greatest in the second place, and the least in the first; and it therefore stands thus:— 20 : 40 :: 15 : the answer 30, which is found by the rule.

The method of valuing such questions by the sliding-rule, when stated, is given under that head.

COMPOUND PROPORTION.

59. Compound Proportion depends entirely on the same principles as simple proportion. For instance, if 2 feet of fir cost 40 pence, what will 6 feet of mahogany cost, 3 feet of mahogany being equal in value to 9 of fir. Here we may find the price of the 6 feet as if they were fir, and it comes out, by the last article, 120 pence, but 3 is to 9 as the price of fir is to that of mahogany; therefore we put the 120, the price of 6 feet of fir, in the third term, and state the proportion, 3 : 9 :: 120 : 360, the price of 6 feet of mahogany.

The same would have been more easily found by stating it thus:

$$\left.\begin{array}{l} 2:\ 6:: \\ 3:\ 9:: \end{array}\right\} 40:$$

$$6:54::\ \ 40:360$$

where the proportions are stated under each other, and multiplied together, which produces $3 \times 2 = 6$ and $6 \times 9 = 54$, two terms of a new proportion, in the simple rule, where 40 is the third term; and this is only the particular example of a general rule, where we may have as many proportions as we please reduced to the form of a simple question in the rule of three. As, therefore, that quantity which is of the same kind with the required answer is put in the third term, the rest will be found to go in pairs; two expressing relation of price, two relation of quality, two relation of time, which must be put in proper order in the first and second terms, as directed for simple proportion. When this is done, all the first terms of these several proportions are to be multiplied together for a new first term, all the second terms together for a new second term, which being placed with the third, in the form of simple proportion, and operated upon as there directed, will give the answer.

Forty boys are set to dig a trench in summer; 14 spadefulls can be dug in summer for 12 in winter; 6 men can do as much as 13 boys; and 16 men can do it in 104 days in winter: how long will the boys take? Here the answer is to be, how many days? We have in the question 104 days; the third term, relative of difficulty, 14 spadefulls and 12 spadefulls; of strength, 6 men to 13 boys; relation of numbers, 16 to 40; which will be stated thus:

$$\left.\begin{array}{lr} \text{Relation of number,} & 40:16 \\ \text{Relation of difficulty,} & 14:12 \\ \text{Relation of strength,} & 6:13 \end{array}\right\} ::104\ \begin{array}{l} \text{makes the time less.} \\ \text{makes the time less.} \\ \text{makes the time greater.} \end{array}$$

Product, ...3360 : 2496 : : 104 :

ARITHMETICAL AND GEOMETRICAL PROPORTIONS AND PROGRESSIONS.

60. The subject of this chapter is often referred to in elementary books on mechanical science; and for this reason, we shall draw the attention of the reader, for a little while, to the subject.

61. When we inquire as to the difference of two numbers, we inquire for their arithmetical ratio; but when we inquire as to the quotient of two numbers, we inquire for their geometrical ratio. Thus $12 - 3 = 9$ and $12 \div 3 = 4$; here 9 is the arithmetical ratio of 12 and 3, and 4 is the geometrical ratio of the same numbers. From this it will be seen, that ratio and relation are terms which have the same signification.

62. When four numbers follow each other, and are such that the difference of the first two is the same as, or equal to, the difference of the last two, these numbers are said to be in arithmetical proportion; thus the numbers 12, 7, 9, 4, form an arithmetical proportion, because the difference of 12 and 7 is the same as the difference of 9 and 4, both being 5. The numbers in an arithmetical proportion may be varied in their position, but still the result will be an arithmetical proportion; for instance, 12, 7, 9, 4, may be written 12, 9, 7, 4, or 9, 12, 4, 7: but the most remarkable property of arithmetical proportions is this, that the sum of the first and last terms is always equal to the sum of the second and third; thus, $12 + 4 = 16$ and $9 + 7 = 16$; and from this it evidently follows, that, to find the fourth term, we add the second and third terms together, and from their sum subtract the first, the remainder is the fourth term.

63. An arithmetical progression is a series of numbers such, that, in taking any three numbers in succession, the

difference of the first and second is the same as the difference of the second and third; thus, 1, 2, 3, 4, 5, 6, 7, 8, or 14, 12, 10, 8, 6, 4, 2, where the differences of the succeeding numbers in the first is 1, and in the second 2. As the numbers in the first increase from the beginning, it is called an increasing arithmetical series, or progression, and as they decrease, in the second example, from the beginning, it is called a decreasing arithmetical progression, or series.

64. Let us place any one of these progressions above itself, in this manner—

2	4	6	8	10	12	14
14	12	10	8	6	4	2
16	16	16	16	16	16	16

Writing the same progression as increasing and decreasing, the respective terms of the one being directly under the respective terms of the other in columns, as above, the lowest line of the three being the sums of the several columns, which are all seen to be 16. Now, it will be obvious, that the first column consists of the first and last terms of the series, 2, 4, 6, &c., with their sum, which is 16; the second column consists of the first but one and the last but one of the terms of the same series, together with their sum, which is likewise 16. The third column consists of the first but two and the last but two terms, with their sum, which again is 16. We may therefore infer, that, in an arithmetical progression, the sum of any two terms, equally distant from the first and last, is equal to the sum of any other two terms which are equally distant from the first and last, or equal to the sum of the first and last. It will also be seen, that the under line, or sum of the two series, is therefore equal to twice the sum of one of the progressions. Now, there are seven 16ns, or 112, which is twice the sum of the progression, therefore 2)112(56 is the sum of the progression.

65. It is also apparent, that if any term be wanting, that

term may be found by adding the common difference, or arithmetical ratio, of the progression, to the term going before the term sought, or subtracting it from the term which follows, if the series is increasing, but the reverse if decreasing. Thus 2, 4, 8, the term awanting between the 4 and 8, may be supplied either by adding the common difference, 2, to the 4, or subtracting it from the 8, and we thus get 6. The same may be found by taking the sum of the terms on each side of the term sought, and dividing by 2; thus, $4+8=12$, then 2) 12 (6, the same as before; so, likewise, 3, 5, 7, 9, 13; to fill up the term awanting between 9 and 13 we have $9+13=22$, therefore 2) 22 (11, which is the number sought, and it is called the arithmetical mean.

66. The quotient of two numbers is their geometrical ratio, and thus a fraction, as $\frac{6}{12}$, expresses the ratio of 6 to 12, and therefore $1:2::6:12$ is the same thing as $\frac{1}{2}=\frac{6}{12}$. We thus get another view of the rule of three, and it is useful to view any subject of this kind in different ways, as by so doing we acquire a more accurate and extensive knowledge of its nature and application. The limits of this book will not permit us to dwell on this subject, as we have discussed the subject of proportion in a former chapter.

67. In a series or progression of numbers, as 2, 4, 8, 16, 32, 64, where the quotient of any term, and that which follows it, is equal to the quotient of any other term, and that which follows it, such progression is said to be geometrical.

68. Let us take the geometrical progression, 2, 6, 18, 54, 162, and write it as we did the arithmetical, both as in increasing and decreasing series, thus—

2	6	18	54	162
162	54	18	6	2
324	324	324	324	324

Here we observe, that the product of the terms of each column is the same, whatever column we take; and we

arrive at a knowledge of the fact, that the product of the first and last terms is the same as the product of any other two terms, one of which is as many places distant from the first as the other is distant from the last term.

69. If one term in the above series were wanting, for instance, the second, that is 6, take the terms on each side of it, and find their product, $2 \times 18 = 36$, now the square root of this, or 6, will be the number sought, which is called the geometrical mean. In like manner we might find the geometrical mean between 18 and 162; thus, $18 \times 162 = 2916$, the square root of which is $2916^{\frac{1}{2}} = 54$, the number sought. The geometrical mean is sometimes called the mean proportional.

70. The sum of any geometrical series may be found thus:

$$\frac{(\text{The greater extreme} \times \text{ratio}) - \text{less extreme,}}{\text{ratio} - 1} = \text{the sum of series.}$$

thus the sum of the last series is—

$$\frac{(162 \times 3) - 2}{3 - 1} = \frac{486 - 2}{2} = \frac{484}{2} = 242, \text{ the sum.}$$

71. Terms relating to proportion often occur in books read by mechanics, of which it would be useful to know the signification; and, to prevent their being misapplied, we give the following illustration. If there be four numbers in proportion, as 4 : 16 :: 3 : 12, then,

Directly, ... 4	:	16	::	3	:	12
Alternately,...... 4	:	3	::	16	:	12
Inversely,........16	:	4	::	12	:	3
{ Compounded, 4 + 16	:	16	::	3 + 12	:	12
{ That is, ...20	:	16	::	15	:	12
{ Divided,.........4 — 16	:	16	::	3 — 12	:	12
{ That is, ...12	:	16	::	9	:	12

D

Converted,......	4	:	16 + 4	::	3	:	12 + 3
That is,..........	4	:	20	::	3	:	15
Also,..............	4	:	16 — 4	::	3	:	12 — 3
That is,..........	4	:	12	::	3	:	9
Mixed,.........	4 + 16	:	4 — 16	::	3 + 12	:	3 — 12
That is,.........	20	:	12	::	15	:	9

To these may be added, duplicate ratio, or ratio of the squares; triplicate ratio, or ratio of the cubes; sub-duplicate ratio, or ratio of the square roots; and sub-triplicate ratio, or ratio of the cube roots.

POSITION.

72. Position is a rule in which, from the assumption of one or more false answers to a problem, the true one is obtained.

73. It admits of two varieties, single position, and double position.

74. In single position the answer is obtained by one assumption; in double position it is obtained by two.

75. Single position may be applied in resolving problems, in which the required number is any how increased or diminished in any given ratio; such as when it is increased or diminished by any part of itself, or when it is multiplied or divided by any number.

76. Double position is used, when the result obtained by increasing or diminishing the required number in a given ratio, is increased or diminished by some number which is no known part of the required number; or when any root or power of the required number, is either directly or indirectly contained in the result given in the question.

SINGLE POSITION.

77. *Rule.*—Assume any number, and perform on it the operations mentioned in the question as being performed on the required number. Then, as the result thus obtained, is to the assumed number, so is the result given in the question, to the number required.

Exam.—Required a number to which if one half, one third, one fourth, and one fifth of itself be added, the sum may be 1644.

Suppose the number to be 60: then, if to 60 one half, one third, one fourth, and one fifth of itself be added, the sum is 137. Hence, according to the rule, as 137 : 1644 :: 60 : 720, the number required. The truth of the result is proved by adding to 720, one half, one third, &c., of itself, and the sum is found to be 1644. The number 60 was here assumed, not as being near the truth, but as being a multiple of 2, 3, 4, and 5; and in this way the operation was kept free from fractions. By the assumption of any other number, however, the answer would have been found correctly, but not so easily. The reason of the operation is so obvious as not to require illustration.

DOUBLE POSITION.

78. *Rule.*—Assume two different numbers, and perform on them separately the operations indicated in the question. Then, as the difference of the results thus obtained, is to the difference of the assumed numbers, so is the difference between the true result and either of the others, to the correction to be applied to the assumed number which gave this result. Add the correction to this number, if the corresponding result was too small; otherwise, subtract it.

79. A more general rule is this: Having assumed two different numbers, perform on them separately the operations indicated in the question, and find the errors of the

results. Then as the difference of the errors, if both results be too great or both too little, or as the sum of the errors, if one result be too great and the other too small, is to the difference of the assumed numbers, so is either error to the correction to be applied to the number that produced that error.

80. When any root or power of the required number is in any way contained in the result given in the question, the preceding rules will only give an approximation to the required number. In this case the assumed numbers should be taken as near the true answer as possible. Then, to approximate the required number still more nearly, assume for a second operation the number found by the first, and that one of the two first assumptions which was nearer the true answer, or any other number that may appear to be nearer it still. In this way, by repeating the operation as often as may be necessary, the true result may be approximated to any assigned degree of accuracy.

81. It may be further observed also, that the method of double position, besides its use in common arithmetic, is of much utility in algebra, affording in many cases a very convenient mode of approximating the roots of equations, and finding the values of unknown quantities in very complicated expressions, without the usual reductions.

82. *Exam.* 1.—Required a number, from which if 2 be subtracted, one third of the remainder will be 5 less than half the required number.

Here, suppose the required number to be 8, from which take 2, and one third of the remainder is 2. This being taken from one half of 8, the remainder is 2, the first result. Suppose, again, the number to be 32, and from it take 2: one third of the remainder is 10, which being taken from the half of 32, the remainder is 6, the second result. Then, the difference of the results being 4, the difference of the assumed numbers 24, and the difference between 5, the true

result, and 6, the result nearest it, being 1; as 4 : 24 :: 1 : 6, the correction to be subtracted from 32, since the result 6 was too great. Hence, the required number is 26.

83. *Exam.* 2.—If one person's age be now only four times as great as another person's, though 7 years ago it was six times as great; what is the age of each?

Here, suppose the age of the younger to be 12 years; then will the age of the older be 48. Take 7 from each of these, and there will remain 5 and 41, their ages 7 years ago. Now, 6 times 5 is 30, which taken from 41, leaves an error of 11 years. By supposing the age of the younger to be 15, and proceeding in a similar manner, the error is found to be 5 years. Hence, as 6, the difference of the errors, (both results being too small,) is to 3, the difference of the assumed numbers, so is 5, the less error, to 2½, the correction; which being added to 15, the sum, 17½, is the age of the younger, and consequently that of the older must be 70.

Both the rules above given for double position depend on the principle, that the differences between the true and the assumed numbers, are proportional to the differences between the result given in the question and the results arising from the assumed numbers. This principle is quite correct in relation to all questions which in algebra would be resolved by simple equations, but not in relation to any others; and hence, when applied to others, it gives only approximations to the true results.—The subject is of too little importance to claim farther illustration in this place.

84. *Exam.* 3.—Required a number to which if twice its square be added, the sum will be 100.

It is easy to see that this number must be between 6 and 7. These numbers being assumed, therefore, the sum of 6 and twice its square is 78, and the sum of 7 and twice its square is 105. Then, as 105—78 : 7—6 :: 105—100 : ·18; which being taken from 7, the remainder, 6·82, is the required number, nearly. To this let twice its square be

added, and the result is 99·8448. Then, as 105—99·8448 : 7—6·82 : : 105—100 : ·1746; which being taken from 7, the remainder is 6·8254, the required number still more nearly; and if the operation were repeated with this and the former approximate answer, the required number would be found true for seven or eight figures.

APPENDIX TO ARITHMETIC,

CONTAINING

TABLES OF WEIGHTS AND MEASURES.

ENGLISH.

AVOIRDUPOIS WEIGHT.

Drams.										
16	=	1 Ounce.								
256	=	16	=	1 Pound.						
7168	=	448	=	28	=	1 Quarter.				
286782	=	1792	=	112	=	4	=	1 Cwt.		
573440	=	35840	=	2240	=	80	=	20	=	1 Ton.

Tons are marked *t.*; hundred-weights, *cwt.*; quarters, *qr.*; pounds, *lb.*; ounces, *oz.*; and drams, *dr.*

TROY WEIGHT.

Grains.						
24	=	1 Penny-weight.				
480	=	20	=	1 Ounce.		
5760	=	240	=	12	=	1 Pound.

Pounds are marked *lb.*; ounces, *oz.*; penny-weights, *dwt.*; and grains, *gr.*

LONG MEASURE.

Barley corns.						
3 =	1 Inch.					
36 =	12 =	1 Foot.				
108 =	36 =	3 =	1 Yard.			
594 =	198 =	16·5 =	5·5 =	1 pole.		
23760 =	7920 =	660 =	220 =	40 =	1 Furlong.	
190080 =	63360 =	5280 =	1760 =	320 =	8 =	1 Mile.

SQUARE MEASURE.

Inches.					
144 =	1 Foot.				
1296 =	9 =	1 Yard.			
39204 =	272¼ =	30¼ =	1 Pole.		
1568160 =	10890 =	1210 =	40 =	1 Rood.	
6272640 =	43560 =	4840 =	160 =	4 =	1 Acre.

SOLID MEASURE.

Inches.		
1728 =	1 Foot.	
46656 =	27 =	1 Yard.

WINE MEASURE.

Pints.							
2 =	1 Quart.						
8 =	4 =	1 Gallon.					
336 =	168 =	42 =	1 Tierce.				
504 =	257 =	63 =	1·5 =	1 Hogshead.			
672 =	336 =	84 =	2 =	1·5 =	1 Puncheon.		
1008 =	504 =	126 =	3 =	2 =	1·5 =	1 Pipe.	
2016 =	1008 =	252 =	6 =	4 =	3 =	2 =	1 Tun.

ALE AND BEER MEASURE.

Pints.								
2 =	1 Quart.							
8 =	4 =	1 Gallon.						
72 =	36 =	2 =	1 Firkin.					
144 =	72 =	18 =	2 =	1 Kilderkin.				
288 =	144 =	36 =	4 =	2 =	1 Barrel.			
432 =	216 =	54 =	6 =	3 =	1·5 =	1 Hogshead.		
576 =	288 =	72 =	8 =	4 =	2 =	1·5 =	1 Puncheon.	
864 =	432 =	108 =	12 =	6 =	3 =	2 =	1·5 =	1 But.

DRY MEASURE.

Pints.							
8 =	1 Gallon.						
16 =	2 =	1 Peck.					
64 =	8 =	4 =	1 Bushel.				
256 =	32 =	16 =	4 =	1 Coom.			
512 =	64 =	32 =	8 =	2 =	1 Quarter.		
2560 =	320 =	160 =	40 =	10 =	5 =	1 Wey.	
5120 =	640 =	320 =	80 =	20 =	10 =	2 =	1 Last.

TIME.

60 seconds = 1 minute, 60 minutes = 1 hour,
24 hours = 1 day, 365¼ days = 1 year, nearly.

THE CIRCLE.

The circle is divided into 360 equal parts, called degrees.

Seconds.				
60 =	1 Minute,			
360 =	60 =	1 Degree,		
32400 =	5400 =	90 =	1 Quadrant,	
129600 =	21600 =	360 =	4 =	1 Circumference.

Degrees, minutes, and seconds, are marked °, ′, ″; as, 4° 5′ 6″—4 degrees, 5 minutes, 6 seconds.

REMARKS ON ENGLISH WEIGHTS AND MEASURES.

Troy weight is used frequently by chemists, and also in weighing gold, silver, and jewels; but all metals, except gold and silver, are weighed by avoirdupois weight.

175 troy pounds are equal to 144 avoirdupois pounds.

175 troy ounces = 192 avoirdupois ounces.

14 oz., 11 dwt., 15½ grs. troy = 1 lb. avoirdupois.

18 dwt., 5½ gr. troy = 1 oz. avoirdupois.

3 miles long measure = 1 league.

$69\frac{1}{15}$ English miles = 60 geographical miles.

1089 Scottish acres = 1369 English acres.

A chaldron of coals in London = 36 bushels, and weighs 3136 lbs. avoirdupois, or nearly 1 ton, 8 cwt.

The ale gallon contains 282 cubic inches, and the wine gallon contains 231 cubic inches,—the wine gallon being to the ale gallon nearly as 1 lb. avoirdupois to 1 lb. troy.

By an Act of Parliament passed in 1824, and carried into execution in 1826, Imperial weights and measures were introduced by this.

The pound troy contains 5760 grains.

The pound avoirdupois contains 7000 grains.

The imperial gallon contains 277·274 cubic inches.

The bushel (dry measure) contains 2218·192 cubic inches.

To find the value of the old in terms of the new, or the reverse, the following table of multipliers is given.

	Dry.	Wine.	Ale.
To convert the old into new ×	0·96943	0·83311	1·01704.
To convert new into old ×	1·03153	1·20032	0·98324.

Examples.—What is the value in imperial measure, of 32 wine gallons old measure?

·83311 × 32 = 26·65952 imperial gallons.

In like manner 4 bushels imperial measure = 1·03153 × 4 = 4·12612 old or Winchester bushels.

FRENCH WEIGHTS AND MEASURES.

OLD SYSTEM.

		English Troy Grains.
The Paris Pound	=	7561
Ounce	=	472·5625
Gros	=	59·0703
Grain	=	·8204

		Eng. Inches.
The Paris Royal Foot of 12 Inches	=	12·7977
The Inch	=	1·0659
The Line, or one-twelfth of an Inch	=	·0074

		Eng. Cubical Feet.
The Paris Cubic Foot.........	=	1·211273
The Cubic Inch	=	·000700

Measure of Capacity.

The Paris pint contains 58·145 English cubical inches, and the English wine pint contains 28·875 cubical inches; or the Paris pint contains 2·0171082 English pints; therefore to reduce the Paris pint to the English, multiply by 2·0171082.

NEW SYSTEM.

MEASURES OF LENGTH.

		English Inches.
Millimetre....................................	=	.03937
Centimetre	=	·39370
Decimetre......................................	=	3·93702
Metre...	=	39·37023
Decametre......................................	=	393·70226

Hecatometre	=	3937·02260
Chiliometre	=	39370·22601
Myriometre	=	393702·26014

		M.	P.	Y.	Ft.	In.
A Decametre is	=	0	0	10	2	9·7
A Hecatometre	=	0	0	109	1	·1
A Chiliometre	=	0	4	213	1	10·2
A Myriometre	=	6	1	156	0	·6

Eight Chiliometres are nearly 5 English miles.

MEASURES OF CAPACITY.

		English Cubic Inches.
Millilitre	=	·06102
Centilitre	=	·61024
Decilitre	=	6·10244
Litre	=	61·10244
Decalitre,	=	610·24429
Hecatolitre	=	6102·44288
Chiliolitre	=	61024·42878
Myriolitre	=	610244·28778

A Litre is nearly $2\frac{1}{8}$ wine pints.

14 Decilitres are nearly 3 wine pints.

A Chiliolitre is a tun, 12·75 wine gallons.

WEIGHTS.

		English Grains.
Milligramme	=	·0154
Centigramme	=	·1544
Decigramme	=	1·5444
Gramme	=	15·4440
Decagramme	=	154·4402
Hecatogramme	=	1544·4023
Chiliogramme (Kilogram)	=	15444·0234
Myriogramme	=	154440·2344

A Decagramme is 6 dwts. 10·44 gr. tr.; or 5·65 dr. avoir.
A Hecatogramme is 3 oz. 8·5 dr. avoir.
A Chiliogramme is 2 lbs. 3 oz. 5 dr. avoir.
A Myriogramme is 22 — 1·15 oz. avoir.
100 Myriogrammes are 1 ton, wanting 32·8 lbs.

AGRARIAN MEASURES.

Are, 1 square Decametre	=	3·95 Perches.
Hecatare	=	2 Acres, 1 Rood, 30·1 Perches.

FIR WOOD.

Decistre, 1-10th Stere	=	3·5315 cub. ft. Eng.
Stere, 1 Cubic Metre	=	35·3150 cub. ft.

DIVISION OF THE CIRCLE.

100 seconds	=	1 minute.
100 minutes	=	1 degree.
100 degrees	=	1 quadrant.
4 quadrants	=	1 circle.

THE ENGLISH DIVISION.

60 seconds	=	1 minute.
60 minutes	=	1 degree.
360 degrees	=	1 circle.

GEOMETRY.

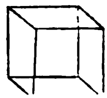
DEFINITIONS.

1. A POINT is that which has position, but no magnitude, nor dimensions; neither length, breadth, nor thickness.

2. A Line is length, without breadth or thickness.

3. A Surface or Superficies, is an extension or a figure of two dimensions, length and breadth; but without thickness.

4. A Body or Solid, is a figure of three dimensions, namely, length, breadth, and depth, or thickness.

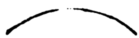


5. Lines are either Right, or Curved, or mixed of these two.

6. A Right Line, or Straight Line, lies all in the same direction, between its extremities; and is the shortest distance between two points.

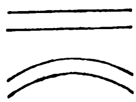
When a line is mentioned simply, it means a Right Line.

7. A Curve continually changes its direction between its extreme points.

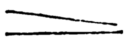


8. Lines are either Parallel, Oblique, Perpendicular, or Tangential.

9. Parallel lines are always at the same perpendicular distance; and they never meet, though ever so far produced.

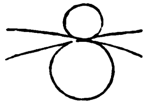


10. Oblique lines change their distance, and would meet, if produced on the side of the least distance.



11. One line is Perpendicular to another, when it inclines not more on the one side than the other, or when the angles on both sides of it are equal.

12. A line or circle is Tangential, or is a Tangent to a circle or other curve, when it touches it, without cutting, when both are produced.



13. An Angle is the inclination or opening of two lines, having different directions, and meeting in a point.

14. Angles are Right or Oblique, Acute or Obtuse.

15. A Right angle is that which is made by one line perpendicular to another. Or when the angles on each side are equal to one another, they are right angles.

16. An Oblique angle is that which is made by two oblique lines; and is either less or greater than a right angle.

17. An Acute Angle is less than a right angle.

18. An Obtuse Angle is greater than a right angle.

19. Superficies are either Plane or Curved.

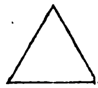
20. A Plane Superficies, or a Plane, is that with which a right line may, every way, coincide. Or, if the line touch the plane in two points, it will touch it in every point. But, if not, it is curved.

21. Plane Figures are bounded either by right lines or curves.

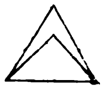
22. Plane figures that are bounded by right lines have names according to the number of their sides, or of their angles; for they have as many sides as angles; the least number being three.

23. A figure of three sides and angles is called a Triangle. And it receives particular denominations from the relations of its sides and angles.

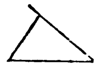
24. An Equilateral Triangle is that whose three sides are all equal.



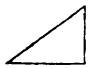
25. An Isosceles Triangle is that which has two sides equal.



26. A Scalene Triangle is that whose three sides are all unequal.



27. A Right-angled Triangle is that which has one right angle.



28. Other triangles are Oblique-angled, and are either obtuse or acute.

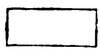
29. An Obtuse-angled Triangle has one obtuse angle.

30. An Acute-angled Triangle has all its three angles acute.

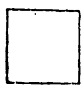
31. A figure of four sides and angles is called a Quadrangle, or a Quadrilateral.

32. A Parallelogram is a quadrilateral which has both its pairs of opposite sides parallel. And it takes the following particular names, viz. Rectangle, Square, Rhombus, Rhomboid.

33. A Rectangle is a parallelogram, having a right angle.

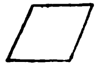


34. A Square is an equilateral rectangle; having its length and breadth equal.



35. A Rhomboid is an oblique-angled parallelogram.

36. A Rhombus is an equilateral rhomboid; having all its sides equal, but its angles oblique.



37. A Trapezium is a quadrilateral which hath not its opposite sides parallel.

38. A Trapezoid has only one pair of opposite sides parallel.

39. A Diagonal is a line joining any two opposite angles of a quadrilateral.

40. Plane figures that have more than four sides are, in general, called Polygons; and they receive other particular names, according to the number of their sides or angles. Thus,

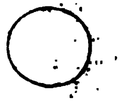
41. A Pentagon is a polygon of five sides; a Hexagon, of six sides; a Heptagon, seven; an Octagon, eight; a Nonagon, nine; a Decagon, ten; an Undecagon, eleven; and a Dodecagon, twelve sides.

42. A Regular Polygon has all its sides and all its angles equal.—If they are not both equal, the polygon is Irregular.

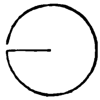
43. An Equilateral Triangle is also a regular figure of three sides, and the Square is one of four: the former being also called a Trigon, and the latter a Tetragon.

44. Any figure is equilateral, when all its sides are equal: and it is equiangular when all its angles are equal. When both these are equal, it is a regular figure.

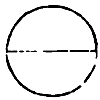
45. A Circle is a plane figure bounded by a curve line, called the Circumference, which is everywhere equidistant from a certain point within, called its Centre.



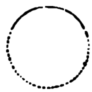
The circumference itself is often called a circle, and also the Periphery.



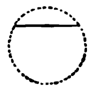
46. The Radius of a circle is a line drawn from the centre to the circumference.



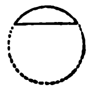
47. The Diameter of a circle is a line drawn through the centre, and terminating at the circumference on both sides.



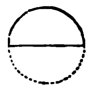
48. An Arc of a circle is any part of the circumference.



49. A Chord is a right line joining the extremities of an arc.

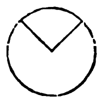


50. A Segment is any part of a circle bounded by an arc and its chord.

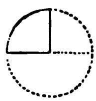


51. A Semicircle is half the circle, or a segment cut off by a diameter.

The half circumference is sometimes called the Semicircle.



52. A Sector is any part of a circle which is bounded by an arc, and two radii drawn to its extremities.

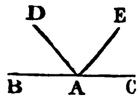


53. A Quadrant, or Quarter of a circle, is a sector having a quarter of the circumference for its arc, and its two radii are perpendicular to each other. A quarter of the circumference is sometimes called a Quadrant.

54. The Height or Altitude of a figure is a perpendicular let fall from an angle, or its vertex, to the opposite side, called the base.

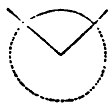
55. In a right-angled triangle, the side opposite the right angle is called the Hypothenuse; and the other two sides are called the Legs, and sometimes the Base and Perpendicular.

56. When an angle is denoted by three letters, of which one stands at the angular point, and the other two on the two sides, that which stands at the angular point is read in the middle.

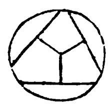


57. The circumference of every circle is supposed to be divided into 360 equal parts, called degrees; and each degree into 60 minutes, each minute into 60 seconds, and so on. Hence a semicircle contains 180 degrees, and a quadrant 90 degrees.

58. The Measure of an angle, is an arc of any circle contained between the two lines which form that angle, the angular point being the centre; and it is estimated by the number of degrees contained in that arc.

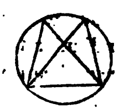


59. Lines, or chords, are said to be Equidistant from the centre of a circle, when perpendiculars drawn to them from the centre are equal.



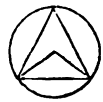
60. And the right line on which the Greater Perpendicular falls, is said to be farther from the centre.

61. An Angle In a Segment is that which is contained by two lines, drawn from any point in the arc of the segment, to the two extremities of that arc.

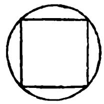


62. An Angle On a segment, or an arc, is that which is contained by two lines, drawn from any point in the opposite or supplemental part of the circumference, to the extremities of the arc, and containing the arc between them.

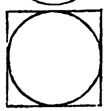
63. An Angle at the circumference, is that whose angular point or summit is any where in the circumference. And an angle at the centre, is that whose angular point is at the centre.



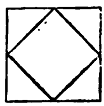
64. A right-lined figure is Inscribed in a circle, or the circle Circumscribes it, when all the angular points of the figure are in the circumference of the circle.



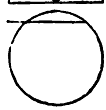
65. A right-lined figure Circumscribes a circle, or the circle is Inscribed in it, when all the sides of the figure touch the circumference of the circle.



66. One right-lined figure is Inscribed in another, or the latter circumscribes the former, when all the angular points of the former are placed in the sides of the latter.



67. A Secant is a line that cuts a circle, lying partly within, and partly without it.



68. Two triangles, or other right-lined figures, are said to be mutually equilateral, when all the sides of the one are equal to the corresponding sides of the other, each to each: and they are said to be mutually equiangular, when the angles of the one are respectively equal to those of the other.

69. Identical figures, are such as are both mutually equilateral and equiangular; or that have all the sides and all the angles of the one, respectively equal to all the sides and all the angles of the other, each to each; so that, if the one

figure were applied to, or laid upon the other, all the sides of the one would exactly fall upon and cover all the sides of the other; the two becoming as it were but one and the same figure.

70. Similar figures, are those that have all the angles of the one equal to all the angles of the other, each to each, and the sides about the equal angles proportional.

71. The Perimeter of a figure, is the sum of all its sides taken together.

72. A Proposition, is something which is either proposed to be done, or to be demonstrated, and is either a problem or a theorem.

73. A Problem, is something proposed to be done.

74. A Theorem, is something proposed to be demonstrated.

75. A Lemma, is something which is premised, or demonstrated, in order to render what follows more easy.

76. A Corollary, is a consequent truth, gained immediately from some preceding truth, or demonstration.

77. A Scholium, is a remark or observation made upon something going before it.

THEOREMS.

1. In the two triangles ABC, DEF, if the side AC be equal to the side DF, and the side BC equal to the side EF, and the angle C equal to the angle F; then will the two triangles be identical, or equal in all respects.—(Fig. 1.)

2. Let the two triangles ABC, DEF, have the angle A equal to the angle D, the angle B equal to the angle E, and the side AB equal to the side DE; then these two triangles will be identical.—(Fig. 1.)

3. If the triangle ABC have the side AC equal to the side BC; then will the angle B be equal to the angle A.—(Fig. 2.)

The line which bisects the vertical angle of an isosceles triangle, bisects the base, and is also perpendicular to it.

Every equilateral triangle, is also equiangular, or has all its angles equal.

4. If the triangle ABC, have the angle A equal to the angle B, it will also have the side AC equal to the side BC.—(Fig. 3.)

Every equiangular triangle is also equilateral.

5. Let the two triangles ABC, ABD, have their three sides respectively, equal, viz. the side AB equal to AB, AC to AD, and BC to BD; then shall the two triangles be identical, or have their angles equal, viz. those angles that are opposite to the equal sides; namely, the angle BAC to the angle BAD, the angle ABC to the angle ABD, and the angle C to the angle D.—(Fig. 4.)

6. Let the line AB meet the line CD; then will the two angles ABC, ABD, taken together, be equal to two right angles.—(Fig. 5.)

7. Let the two lines AB, CD, intersect in the point E; then will the angle AEC be equal to the angle BED, and the angle AED equal to the angle CEB.—(Fig. 6.)

8. Let ABC be a triangle, having the side AB produced to D; then will the outward angle CBD be greater than either of the inward opposite angles A or C.—(Fig. 7.)

9. Let ABC be a triangle, having the side AB greater than the side AC; then will the angle ACB, opposite the greater side AB, be greater than the angle B, opposite the less side AC.—(Fig. 8.)

10. Let ABC be a triangle; then will the sum of any two of its sides be greater than the third side; as, for instance, AC + CB greater than AB.—(Fig. 9.)

11. Let ABC be a triangle; then will the difference of any

two sides, as AB — AC, be less than the third side BC.—(Fig. 10.)

12. Let the line EF cut the two parallel lines AB, CD; then will the angle AEF be equal to the alternate angle EFD.—(Fig. 11.)

13. Let the line EF, cutting the two lines AB, CD, make the alternate angles AEF, DFE, equal to each other; then will AB be parallel to CD.—(Fig. 12.)

14. Let the line EF cut the two parallel lines AB, CD; then will the outward angle EGB be equal to the inward opposite angle GHD, on the same side of the line EF; and the two inward angles BGH, GHD, taken together, will be equal to two right angles.—(Fig. 13.)

15. Let the lines AB, CD, be each of them parallel to the line EF; then shall the lines AB, CD, be parallel to each other.—(Fig. 14.)

16. Let the side AB, of the triangle ABC, be produced to D; then will the outward angle CBD be equal to the sum of the two inward opposite angles A and C.—(Fig. 15.)

17. Let ABC be any plane triangle; then the sum of the three angles $A + B + C$ is equal to two right angles.—(Fig. 16.)

If two angles in one triangle, be equal to two angles in another triangle, the third angles will also be equal, and the two triangles equiangular.

If one angle in one triangle, be equal to one angle in another, the sums of the remaining angles will also be equal.

If one angle of a triangle be right, the sum of the other two will also be equal to a right angle, and each of them singly will be acute, or less than a right angle.

The two least angles of every triangle are acute, or each less than a right angle.

18. Let ABCD be a quadrangle; then the sum of the four inward angles, $A + B + C + D$ is equal to four right angles.—(Fig. 17.)

19. Let ABCDE be any figure; then the sum of all its inward angles, $A + B + C + D + E$, is equal to twice as many right angles, wanting four, as the figure has sides.—(Fig. 18.)

20. Let A, B, C, &c., be the outward angles of any polygon, made by producing all the sides; then will the sum $A + B + C + D + E$, of all those outward angles, be equal to four right angles.—(Fig. 19.)

21. If AB, AC, AD, &c., be lines drawn from the given point A, to the indefinite line DE, of which AB is perpendicular; then shall the perpendicular AB be less than AC, and AC less than AD, &c.—(Fig. 20.)

22. Let ABCD be a parallelogram, of which the diagonal is BD; then will its opposite sides and angles be equal to each other, and the diagonal BD will divide it into two equal parts, or triangles.—(Fig. 21.)

If one angle of a parallelogram be a right angle, all the other three will also be right angles, and the parallelogram a rectangle.

The sum of any two adjacent angles of a parallelogram is equal to two right angles.

23. Let ABCD be a quadrangle, having the opposite sides equal, namely, the side AB equal to DC, and AD equal to BC; then shall these equal sides be also parallel, and the figure a parallelogram.—(Fig. 21.)

24. Let AB, DC, be two equal and parallel lines; then will the lines AD, BC, which join their extremes, be also equal and parallel.—(Fig. 21.)

25. Let ABCD, ABEF, be two parallelograms, and ABC, ABF, two triangles, standing on the same base AB, and between the same parallels AB, DE; then will the parallelogram ABCD be equal to the parallelogram ABEF,

and the triangle ABC equal to the triangle ABF.—(Fig. 22.)

Parallelograms, or triangles, having the same base and altitude, are equal. For the altitude is the same as the perpendicular or distance between the two parallels, which is everywhere equal, by the definition of parallels.

Parallelograms, or triangles, having equal bases and altitudes, are equal. For, if the one figure be applied with its base on the other, the bases will coincide or be the same, because they are equal: and so the two figures, having the same base and altitude, are equal.

26. Let ABCD be a parallelogram, and ABE a triangle, on the same base AB, and between the same parallels AB, DE; then will the parallelogram ABCD be double the triangle ABE, or the triangle half the parallelogram.—(Fig. 23.)

A triangle is equal to half a parallelogram of the same base and altitude, because the altitude is the perpendicular distance between the parallels, which is everywhere equal, by the definition of parallels.

If the base of a parallelogram be half that of a triangle, of the same altitude, or the base of the triangle be double that of the parallelogram, the two figures will be equal to each other.

27. Let BD, FH, be two rectangles, having the sides AB, BC, equal to the sides EF, FG, each to each; then will the rectangle BD be equal to the rectangle FH.—(Fig. 24.)

28. Let AC be a parallelogram, BD a diagonal, EIF parallel to AB or DC, and GIH parallel to AD or BC, making AI, IC, complements to the parallelograms EG, HF, which are about the diagonal DB: then will the complement AI be equal to the complement IC.—(Fig. 25.)

29. Let AD be the one line, and AB the other, divided into the parts AE, EF, FB; then will the rectangle contained by AD and AB, be equal to the sum of the rectangles of AD

and AE, and AD and FB: thus expressed, $AD \cdot AB = AD \cdot AE + AD \cdot EF + AD \cdot FB$.*—(Fig. 26.)

If a right line be divided into any two parts, the square on the whole line, is equal to both the rectangles of the whole line and each of the parts.

30. Let the line AB be the sum of any two lines AC, CB; then will the square of AB be equal to the squares of AC, CB, together with twice the rectangle of AC . CB. That is, $AB^2 = AC^2 + CB^2 + 2AC \cdot CB$.—(Fig. 27.)

If a line be divided into two equal parts; the square of the whole line will be equal to four times the square of half the line.

31. Let AC, BC, be any two lines, and AB their difference; then will the square of AB be less than the squares of AC, BC, by twice the rectangle of AC and BC. Or, $AB^2 = AC^2 + BC^2 - 2AC \cdot BC$.—(Fig. 28.)

32. Let AB, AC, be any two unequal lines; then will the difference of the squares of AB, AC, be equal to a rectangle under their sum and difference. That is, $AB^2 - AC^2 = \overline{AB + AC} \cdot \overline{AB - AC}$.—(Fig. 29.)

33. Let ABC be a right-angled triangle, having the right angle C; then will the square of the hypothenuse AB, be equal to the sum of the squares of the other two sides AC, CB. Or $AB^2 = AC^2 + BC^2$.—(Fig. 30.)

34. Let ABC be any triangle, having CD perpendicular to AB; then will the difference of the squares of AC, BC, be equal to the difference of the squares of AD, BD; that is, $AC^2 - BC^2 = AD^2 - BD^2$.—(Fig. 31.)

* Instead of the mark $\times$, a point is often used; thus, length $\times$ breadth = area, is the same as length . breadth = area. Instead of the parenthesis, a stroke is often used; thus, (first + last) $\div$ 2 = arithmetical mean, is the same thing as $\overline{\text{first} + \text{last}} \div 2$ = arithmetical mean. For the square root this mark $\sqrt{\ }$ is sometimes used, and for the cube root $\sqrt[3]{\ }$, &c.

35. Let ABC be a triangle, obtuse-angled at B, and CD perpendicular to AB; then will the square of AC be greater than the squares of AB, BC, by twice the rectangle of AB, BD. That is, $AC^2 = AB^2 + BC^2 + 2AB \cdot BD$.—(Fig. 31.)

36. Let ABC be a triangle, having the angle A acute, and CD perpendicular to AB; then will the square of BC, be less than the squares of AB, AC, by twice the rectangle of AB, AD. That is, $BC^2 = AB^2 + AC^2 - 2AD \cdot AB$.—(Fig. 31.)

37. Let ABC be a triangle, and CD the line drawn from the vertex to the middle of the base AB, bisecting it into the two equal parts AD, DB; then will the sum of the squares of AC, CB, be equal to twice the sum of the squares of CD, AD; or $AC^2 + CB^2 = 2CD^2 + 2AD^2$.—(Fig. 32.)

38. Let ABC be the isosceles triangle, and CD a line drawn from the vertex to any point D in the base: then will the square of AC, be equal to the square of CD, together with the rectangle of AD and DB. That is, $AC^2 = CD^2 + AD \cdot DB$.—(Fig. 33.)

39. Let ABCD be a parallelogram, whose diagonals intersect each other in E: then will AE be equal to EC, and BE to ED; and the sum of the squares of AC, BD, will be equal to the sum of the squares of AB, BC, CD, DA. That is,

$$AE = EC, \text{ and } BE = ED,$$

and $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$.—(Fig. 34.)

40. Let AB be any chord in a circle, and CD a line drawn from the centre C to the chord. Then, if the chord be bisected in the point D, CD will be perpendicular to AB.—(Fig. 35.)

41. Let ABC be a circle, and D a point within it; then if any three lines, DA, DB, DC, drawn from the point D to the circumference, be equal to each other, the point D will be the centre.—(Fig. 36.)

42. Let the two circles ABC, ADE, touch one another inter-

nally in the point A; then will the point A and the centres of those circles be all in the same right line.—(Fig. 37.)

43. Let the two circles ABC, ADE, touch one another externally at the point A; then will the point of contact A and the centres of the two circles be all in the same right line.—(Fig. 38.)

44. Let AB, CD, be any two chords at equal distances from the centre G; then will these two chords AB, CD, be equal to each other.—(Fig. 39.)

45. Let the line ADB be perpendicular to the radius CB of a circle; then shall AB touch the circle in the point D only.—(Fig. 40.)

46. Let AB be a tangent to a circle, and CD a chord drawn from the point of contact C; then is the angle BCD measured by half the arc CFD, and the angle ACD measured by half the arc CGD.—(Fig. 41.)

47. Let BAC be an angle at the circumference; it has for its measure, half the arc BC which subtends it.—(Fig. 42.)

48. Let C and D be two angles in the same segment ACDB, or, which is the same thing, standing on the supplemental arc AEB; then will the angle C be equal to the angle D.—(Fig. 43.)

49. Let C be an angle at the centre C, and D an angle at the circumference, both standing on the same arc or same chord AB; then will the angle C be double of the angle D, or the angle D equal to half the angle C.—(Fig. 44.)

50. If ABC or ADC be a semicircle; then any angle D in that semicircle, is a right angle.—(Fig. 45.)

51. If AB be a tangent, and AC a chord, and D any angle in the alternate segment ADC; then will the angle D be equal to the angle BAC made by the tangent and chord of the arc AEC.—(Fig. 46.)

52. Let ABCD be any quadrilateral inscribed in a circle;

then shall the sum of the two opposite angles A and C, or B and D, be equal to two right angles.—(Fig. 47.)

53. If the side AB, of the quadrilateral ABCD, inscribed in a circle, be produced to E; the outward angle DAE will be equal to the inward opposite angle C.—(Fig. 48.)

54. Let the two chords AB, CD, be parallel; then will the arcs AC, BD, be equal; or AC $=$ BD.—(Fig. 49.)

55. Let the tangent ABC be parallel to the chord DF; then are the arcs BD, BF, equal; that is, BD $=$ BF.—(Fig. 50.)

56. Let the two chords AB, CD, intersect at the point E; then the angle AEC, or DEB, is measured by half the sum of the two arcs AC, DB.—.(Fig. 51.)

57. Let the angle E be formed by two secants EAB and ECD; this angle is measured by half the difference of the two arcs AC, DB, intercepted by the two secants.—(Fig. 52.)

58. Let EB, ED, be two tangents to a circle at the points A, C; then the angle E is measured by half the difference of the two arcs CFA, CGA.—(Fig. 53.)

59. Let the two lines AB, CD, meet each other in E; then the rectangle of AE, EB, will be equal to the rectangle of CE, ED. Or, AE . EB $=$ CE . ED.—(Fig. 54.)

When one of the lines in the second case, as DE, by revolving about the point E, comes into the position of the tangent EC or ED, the two points C and D running into one; then the rectangle of CE, ED, becomes the square of CE, because CE and DE are then equal. Consequently, the rectangle of the parts of the secant, AE . EB, is equal to the square of the tangent, CE^2.—(Fig. 55.)

60. Let ABC, DEF, be two equiangular triangles, having the angle A $=$ the angle D, the angle B $=$ the angle E, and the angle C $=$ the angle F; also, the like sides AB, DE, and AC, DF, being those opposite the equal angles: then will the rectangle of AB, DF, be equal to the rectangle of AC, DE.—(Fig. 56.)

61. Let CD be the perpendicular, and CE the diameter of the circle about the triangle ABC; then the rectangle CA . CB is = the rectangle CD . CE.—(Fig. 57.)

62. Let CD bisect the angle C of the triangle ABC; then the square CD^2 + the rectangle AD . DB is = the rectangle AC . CB.—(Fig. 58.)

63. Let ABCD be any quadrilateral inscribed in a circle, and AC, BD, its two diagonals; then the rectangle AC . BD is = the rectangle AB . DC + the rectangle AD . BC.—(Fig. 59.)

64. Let the two triangles ADC, DEF, have the same altitude, or be between the same parallels AE, CE; then is the surface of the triangle ADC, to the surface of the triangle DEF, as the base AD is to the base DE. Or, AD : DE :: the triangle ADC : the triangle DEF.—(Fig. 60.)

65. Let ABC, BEF, be two triangles having the equal bases AB, BE, and whose altitudes are the perpendiculars CG, FH; then will the triangle ABC : the triangle BEF :: CG : FH.—(Fig. 61.)

Triangles and parallelograms, when their bases are equal, are to each other as their altitudes; and by the foregoing one, when their altitudes are equal, they are to each other as their bases; therefore, universally, when neither are equal, they are to each other in the compound ratio, or as the rectangle or product of their bases and altitudes.

66. Let the four lines A, B, C, D, be proportionals, or A : B :: C : D; then will the rectangle of A and D be equal to the rectangle of B and C; or the rectangle A . D = B . C.—(Fig. 62.)

67. Let DE be parallel to the side BC of the triangle ABC; then will AD : DB :: AE : EC.

AB : AC :: AD : AE,
AB : AC :: BD : CE.—(Fig. 63.)

68. Let the angle ACB, of the triangle ABC, be bisected by the line CD, making the angle r equal to the angle s: then will the segment AD be to the segment DB, as the side AC is to the side CB. Or, AD : DB :: AC : CB.—(Fig. 64.)

69. In the two triangles ABC, DEF, if AB : DE :: AC : DF :: BC : EF; the two triangles will have their corresponding angles equal.—(Fig. 65.)

70. Let ABC, DEF, be two triangles, having the angle A = the angle D, and the sides AB, AC, proportional to the sides DE, DF; then will the triangle ABC be equiangular with the triangle DEF.—(Fig. 66.)

71. Let ABC be a right-angled triangle, and CD a perpendicular from the right angle C to the hypothenuse AB; then will

CD be a mean proportional between AD and DB;
AC a mean proportional between AB and AD;
BC a mean proportional between AB and BD.

(Fig. 67.)

72. All similar figures are to each other, as the squares of their like sides.

73. Similar figures inscribed in circles, have their like sides, and also their whole perimeters, in the same ratio as the diameters of the circles in which they are inscribed.

74. Similar figures inscribed in circles, are to each other as the squares of the diameters of those circles.

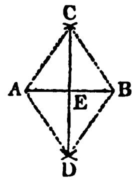
75. The circumferences of all circles are to each other as their diameters.

76. The areas or spaces of circles, are to each other as the squares of their diameters, or of their radii.

77. The area of any circle, is equal to the rectangle of half its circumference and half its diameter.

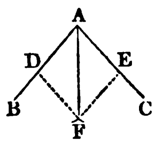
PROBLEMS.

1. *To bisect a line* AB; *that is, to divide it into two equal parts.*



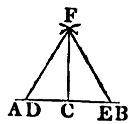
From the two centres A and B, with any equal radii, describe arcs of circles, intersecting each other in C and D; and draw the line CD, which will bisect the given line AB in the point E.

2. *To bisect an angle* BAC.



From the centre A, with any radius, describe an arc cutting off the equal lines AD, AE; and from the two centres D, E, with the same radius, describe arcs intersecting in F; then draw AF, which will bisect the angle A as required.

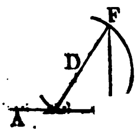
3. *At a given point* C, *in a line* AB, *to erect a perpendicular.*



From the given point C, with any radius, cut off any equal parts CD, CE, of the given line; and, from the two centres D and E, with any one radius, describe arcs intersecting in F; then join CF, which will be perpendicular as required.

OTHERWISE.

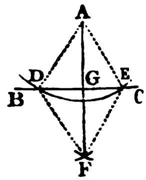
When the given point C *is near the end of the line.*



From any point D, assumed above the line, as a centre, through the given point C describe a circle, cutting the given line at E; and through E and the centre D, draw the diameter EDF; then join CF, which will be the perpendicular required.

4. *From the given point* A, *to let fall a perpendicular on a given line* BC.

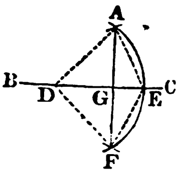
From the given point A as a centre, with any convenient radius, describe an arc, cutting the given line at the two points D and E; and from the two centres D, E, with any radius, describe two arcs, intersecting at F; then draw AGF, which will be perpendicular to BC as required.



OTHERWISE.

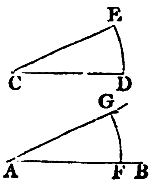
When the given point is nearly opposite the end of the line.

From any point D, in the given line BC, as a centre, describe the arc of a circle through the given point A, cutting BC in E; and from the centre E, with the radius EA, describe another arc, cutting the former in F; then draw AGF, which will be perpendicular to BC as required.



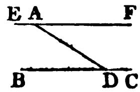
5. *At a given point* A, *in a line* AB, *to make an angle equal to a given angle* C.

From the centres A and C, with any one radius, describe the arcs DE, FG. Then, with radius DE, and centre F, describe an arc, cutting FG in G. Through G draw the line AG, and it will form the angle required.



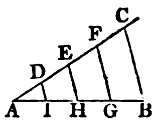
6. *Through a given point* A, *to draw a line parallel to a given line* BC.

From the given point A draw a line AD to any point in the given line BC. Then draw the line EAF making the angle at A equal to the angle at D (by prob. 5); so shall EF be parallel to BC as required.



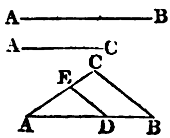
7. *To divide a line* AB *into any proposed number of equal parts.*

Draw any other line AC, forming any angle with the given line AB; on which set off as many of any equal parts AD, DE, EG, FC, as the line AB is to be divided into. Join BC; parallel to which draw the other lines FG, EH, DI; then these will divide AB in the manner as required.—For those parallel lines divide both the sides AB, AC, proportionally.



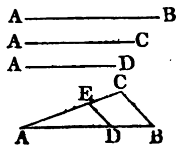
8. *To find a third proportional to two given lines* AB, AC.

Place the two given lines AB, AC, forming any angle at A; and in AB take also AD equal to AC. Join BC, and draw DE parallel to it; so will AE be the third proportional sought.

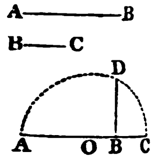


9. *To find a fourth proportional to three lines* AB, AC, AD.

Place two of the given lines AB, AC, making any angle at A; also place AD on AB. Join BC; and parallel to it draw DE; so shall AE be the fourth proportional as required.

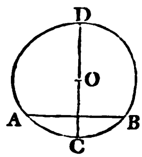


10. *To find a mean proportional between two lines* AB, BC.



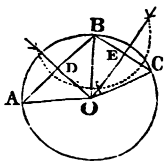
Place AB, BC, joined in one straight line AC: on which, as a diameter, describe the semicircle ADC; to meet which erect the perpendicular BD; and it will be the mean proportional sought, between AB and BC.

11. *To find the centre of a circle.*



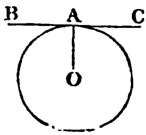
Draw any chord AB; and bisect it perpendicularly with the line CD, which will be a diameter. Therefore CD bisected in O, will give the centre, as required.

12. *To describe the circumference of a circle through three given points* A, B, C.



From the middle point B draw chords BA, BC, to the two other points, and bisect these chords perpendicularly by lines meeting in O, which will be the centre. Then from the centre O, at the distance of any one of the points, as OA, describe a circle, and it will pass through the two other points B, C, as required.

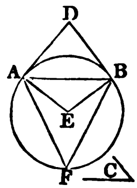
13. *To draw a tangent to a circle, through a given point* A.



When the given point A is in the circumference of the circle, join A and the centre O; perpendicular to which draw BAC, and it will be the tangent.

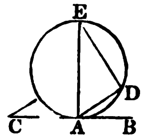
14. *On a given line* B *to describe a segment of a circle, to contain a given angle* C.

At the ends of the given line make angles DAB, DBA, each equal to the given angle C. Then draw AE, BE, perpendicular to AD, BD; and with the centre E, and radius EA or EB, describe a circle; so shall AFB be the segment required, as any angle F made in it will be equal to the given angle C.



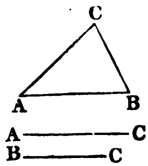
15. *To cut off a segment from a circle, that shall contain a given angle* C.

Draw any tangent AB to the given circle; and a chord AD to make the angle DAB equal to the given angle C; then DEA will be the segment required, any angle E made in it being equal to the given angle C.



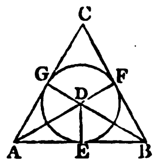
16. *To make a triangle with three given lines*, AB, AC, BC.

With the centre A, and distance AC, describe an arc. With the centre B, and distance BC, describe another arc, cutting the former in C. Draw AB, BC, and ABC will be the triangle required.



17. *To inscribe a circle in a given triangle* ABC.

Bisect any two angles A and B, with the two lines AD, BD. From the intersection D, which will be the centre of the circle, draw the perpendiculars DB, DF, DG, and they will be the radii of the circle required.

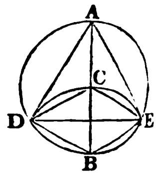


18. *To describe a circle about a given triangle* ABC.

Bisect any two sides with two of the perpendiculars DE, DF, DG, and D will be the centre.—(Fig. 68.)

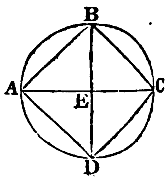
19. *To inscribe an equilateral triangle in a given circle.*

Through the centre C draw any diameter AB. From the point B as a centre, with the radius BC of the given circle, describe an arc DCE. Join AD, AE, DE, and ADE is the equilateral triangle sought.



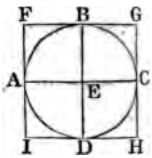
20. *To inscribe a square in a given circle.*

Draw two diameters AC, BD, crossing at right angles in the centre E. Then join the four extremities A, B, C, D, with right lines, and these will form the inscribed square ABCD.



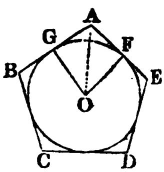
21. *To describe a square about a givencircle.*

Draw two diameters AC, BD, crossing at right angles in the centre E. Then through their four extremities draw FG, IH, parallel to AC, and FI, GH, parallel to BD, and they will form the square FGHI.

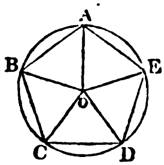


22. *To inscribe a circle in a regular polygon.*

Bisect any two sides of the polygon by the perpendiculars GO, FO, and their intersection O will be the centre of the inscribed circle, and OG or OF will be the radius.

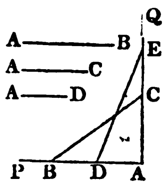


23. *To describe a circle about a regular polygon.*



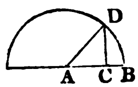
Bisect any two of the angles, C and D, with the lines CO, DO; then their intersection O will be the centre of the circumscribing circle; and OC, or OD, will be the radius.

24. *To make a square equal to the sum of two or more given squares.*



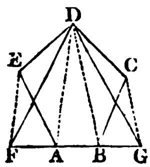
Let AB and AC be the sides of two given squares. Draw two indefinite lines AP, AQ, at right angles to each other; in which place the sides AB, AC, of the given squares; join BC; then a square described on BC will be equal to the sum of the two squares described on AB and AC.

25. *To make a square equal to the difference of two given squares.*



Let AB and AC, taken in the same straight line, be equal to the sides of the two given squares. From the centre A, with the distance AB, describe a circle; and make CD perpendicular to AB, meeting the circumference in D; so shall a square described on CD be equal to $AD^2 - AC^2$, or $AB^2 - AC^2$, as required.

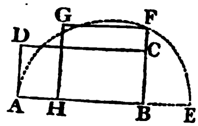
26. *To make a triangle equal to a given pentagon* ABCDE.



Draw DA and DB, and also EF, CG, parallel to them, meeting AB produced at F and G: then draw DF and DG; so shall the triangle DFG be equal to the given pentagon ABCDE.

27. *To make a square equal to a given rectangle* ABCD.

Produce one side AB, till BE be equal to the other side BC. On AE as a diameter describe a circle, meeting BC produced at F: then will BF be the side of the square BFGH, equal to the given rectangle BD, as required.



APPENDIX TO GEOMETRY.

INSTRUMENTS.

28. To facilitate the construction of geometrical figures, we add a short description of a few useful instruments which do not belong to the common pocket-case.

29. There is, first, a flat ruler, AB fig. 69, from one to two feet in length, for which the common Gunter's scale may be substituted; and, secondly, a triangular piece of wood, *a*, *b*, *c*, flat, and about the same thickness as the ruler: the sides *ab* and *bc* are equal to one another, and form a right angle at *b*. For the convenience of sliding, there is usually a hole in the middle of the triangle, as may be seen in the figure.

30. By means of these simple instruments many very useful geometrical problems may be performed. Thus, to draw a line through a given point parallel to a given line. Lay the triangle on the paper so that one of its sides will coincide with the given line to which the parallel is to be drawn; then, keeping the triangle steady, lay the ruler on the paper, with its edge applied to either of the other sides of the tri-

angle; then, keeping the ruler firm, move the triangle along its edge, up or down, to the given point; the side of the triangle which was placed on the given line will always keep parallel to itself, and hence a parallel may be drawn through the given point.

31. To erect a perpendicular on a given line, and from any given point in that line, we have only to apply the ruler to the given line, and place the triangle so, that its right angle shall touch the given point in the line, and one of the sides about the right angle, placed to the edge of the ruler—the other side will give the perpendicular required.

32. If the given point be either above or below the line, the process is equally easy. Place one of the sides of the triangle about the right angle on the given line, and the ruler on the side opposite the right angle, then slide the triangle on the edge of the ruler till the given point from which the perpendicular is to be drawn is on the other side, then this side will give the perpendicular.

33. Other problems may be performed with these instruments, the method of doing which it will be easy for the reader to contrive for himself.

34. When arcs of circles of great diameter are to be drawn, the use of a compass may be substituted by a very simple contrivance. Draw the chord of the arc to be described, and place a pin at each extremity, A and B, fig. 70; then place two rulers jointed at C, and forming an angle, ACB = the supplement of half the given number of degrees; that is to say, the number of degrees which the arc whose chord given is to contain, is to be halfed, and this half being subtracted from 180 degrees, will give the degrees which form the angle at which the rulers are placed, that is the angle ACB. This being done, the edges of the rulers are moved along against the pins, and a pencil at C will describe the arc required.

35. Large circles may be described by a contrivance

equally simple. On an axle, a foot or a foot and a half long, there are placed two wheels, M and F, fig. 71, of which one is fixed to the axle, namely M, and the other is capable of being shifted to different parts of the axle, and, by means of a thumb-screw, made capable of being fixed at any point on the axle. These wheels are of different diameters, say of 3 and 6 inches, the fixed wheel F being the largest. This instrument being moved on the paper, the circles M and F will roll, and describe circles of different radii: the axle will always point to the centre of these circles, and there will be this proportion:

As the diameter of the large wheel
Is to the difference of the diameters of the two wheels,
So is the radius of the circle to be described by the large wheel
To the distance of the two wheels on the axle.

36. If the diameters of the wheels are as above stated, and it is required to describe a circle of 3 feet radius, then from the above proportion we have 6 : 6 — 3 : : 3 feet or 36 inches : 18 inches = the distance of the two wheels, to describe a circle 6 feet in diameter.

37. It may be observed, that it will be best to make the difference of the wheels greater if large circles are to be described, as then a shorter instrument will serve the purpose.

38. We will conclude this appendix, by making a few remarks on the Diagonal Scale and Sector, the great use of the latter of which, especially, is seldom explained to the young mechanic.

39. The diagonal scale to be found on the plain scale in common pocket-cases of instruments, is a contrivance for measuring very small divisions of lines; as, for instance, hundredth parts of an inch.

40. Suppose fig. 72 to represent an enlarged view of two divisions of the diagonal scale, and the bottom and top lines

to be divided into two parts, each representing the tenth part of an inch. Now, the perpendicular lines BC, AD, are each divided into ten equal parts, which are joined by the crossing lines 1, 2, 3, 4, &c., and the diagonals BF, DE, are drawn as in the figure. Now, as the division FC is the tenth part of an inch, and as the line FB continually approaches nearer and nearer to BC, till it meets it in B, it will follow, that the part of the line 1 cut off by this diagonal will be a tenth part of FC, because BI is only one-tenth part of BC; so, likewise, 2 will represent two-tenth parts, 3, three-tenth parts, and so on to 9, which is nine-tenth parts, and 10, ten-tenth parts, or the whole tenth of an inch; so that, by means of this diagonal, we arrive at divisions equal to tenth parts of tenth parts of an inch, or hundredths of an inch. With this consideration, an examination of the scale itself will easily show the whole matter. It may be observed, that if half an inch and the quarter of an inch be divided, in the same manner, into tenths and tenths of tenths, we may get thus two-hundredth and four-hundredth parts of an inch.

THE SECTOR.

41. This very useful instrument consists of two equal rulers, each six inches long, joined together by a brass folding joint. These rulers are generally made of boxwood or ivory; and on the face of the instrument, several lines or scales are engraven. Some of these lines or scales proceed from the centre of the joint, and are called *sectorial lines*, to distinguish them from others which are drawn parallel to the edge of the instrument, similar to those on the common Gunter's scale.

42. The sectorial lines are drawn twice on the same face of the instrument; that is to say, each line is drawn on both legs. Those on each face are,

A scale of equal parts, marked	L,
A line of chords, marked	C,
A line of secants, marked	S,
A line of polygons, marked	P, or Pol.

These sectorial lines are marked on one face of the instrument; and on the other there are the following:

A line of sines, marked	S,
A line of tangents, marked	T,
A line of tangents to a less radius, marked	*t*.

This last line is intended to supply the defect of the former, and extends from about 45 to 75 degrees.

43. The lines of chords, sines, tangents, and secants, but not the line of polygons, are numbered from the centre, and are so disposed as to form equal angles at the centre; and it follows from this, that at whatever distance the sector is opened, the angles which the lines form, will always be respectively equal. The distance, therefore, between 10 and 10, on the two lines marked L, will be equal to the distance of 60 and 60 on the two lines of chords, and also to 90 and 90 on the two lines of sines, &c., at any particular opening of the sector.

44. Any extent measured with a pair of compasses, from the centre of the joint to any division on the sectorial lines, is called a *lateral distance*; and any extent taken from a point in a line on the one leg, to the like point on the similar line on the other leg, is called a *transverse* or *parallel distance*.

With these remarks, we shall now proceed to explain the use of the sector, in so far as it is likely to be serviceable to Mechanics.

USE OF THE LINE OF LINES.

45. This line, as was before observed, is marked L, and its uses are,

To Divide a line into any number of equal parts: Take the length of the line by the compasses, and placing one of the points on that number in the line of sines which denotes the number of parts into which the given line is to be divided, open the sector till the other point of the compasses touches the same division on the line of lines marked on the other leg; then, the sector being kept at the same width, the distance from 1 on the line L on the one leg, to 1 on the line L on the other, will give the length of one of the equal divisions of the given line to be divided. Thus, to divide a given line into seven equal parts:—take the length of the given line with the compasses, and setting one point on 7, on the line L of one of the legs, move the other leg out until the other point of the compasses touch 7 on the line L of that leg; this may be called the transverse distance of 7 on the line of lines. Now, keeping the sector at the same opening, the transverse distance of 1 will be the length of one of the 7 equal divisions of the given line; the transverse distance of 2 will be two of these divisions, &c.

46. It will sometimes happen, that the line to be divided will be too long for the largest opening of the sector; and in this case we take the half, or third, or fourth of the line, as the case may be; then the transverse distance of 1 to 1, will be a half, a third, or a fourth of the required equal part.

47. To divide a given line into any number of parts that shall have a certain relation or proportion to each other: Take the length of the whole line to be divided, and placing one point of the compasses at the division on the line of lines on one leg of the sector, at that division which expresses the sum of the parts into which the given line is to

be divided, and open the sector till the other point of the compasses is on the corresponding division on the line of lines of the other leg. This is evidently making the sum of the parts into which the given line is to be divided a transverse distance; and when this is done, the proportional parts will be found by taking, with the same opening of the sector, the transverse distances of the parts required.—To divide a given line into three parts, in the proportion of 2, 3, 4: The sum of these is 9; make the given line a transverse distance between 9 and 9 on the two lines of lines; then the transverse distances of the several numbers 2, 3, 4, will give the proportional parts required.

48. To find a fourth proportional to three given lines: Take the lateral distance of the second, and make it the transverse distance of the first, then will the transverse distance of the third be the lateral distance of the fourth; then, let there be given 6 : 3 :: 8,—make the lateral distance of 3 the transverse distance of 6; then will the transverse distance of 8 be the lateral distance of 4, the fourth proportional required.

49. This sector will be found highly serviceable in drawing plans. For instance, if it is wished to reduce the drawing of a steam engine from a scale of $1\frac{1}{2}$ inches to the foot, to another of $\frac{5}{8}$ to the foot. Now, in $1\frac{1}{2}$ inches there are $\frac{12}{8}$ parts; so that the drawing will be reduced in the proportion of 12 to 5. Take the lateral distance of 5, and keep the compasses at this opening; then open the sector till the points of the compasses mark the transverse distance of 12; keep now the sector at this opening, and any measure taken on the drawing, to be copied and laid off on the sector as a lateral distance,—the transverse distance taken from that point will give the corresponding measure to be laid down in the new drawing.

50. If the length of the side of a triangle, of which we have the drawing, is to be reckoned 45; what are the lengths

of the other two sides? Take the length of the side given, by the compasses, and open the sector till the measure be the transverse distance of 45 to 45; then the lengths of the other sides being applied transversely, will give their numerical lengths.

USE OF THE LINE OF CHORDS.

51. By means of the sector, we may dispense with the protractor. Thus, to lay down an angle of any number of degress:—take the radius of the circle on the compasses, and open the sector till this becomes the transverse distance of 60 on the line of chords; then take the transverse distance of the required number of degrees, keeping the sector at the same opening; and this transverse distance being marked off on an arc of the circle whose radius was taken, will be the required number of degrees.

We will not enter farther on the use of the sectoral lines, as what we have said will, we hope, be found sufficient for the purposes of the practical mechanic.

MECHANICAL DRAWING.

52. A FLAT rectangular board is first to be provided, of any convenient size, as from 18 to 30 inches long, and from 16 to 24 inches broad. It may be made of fir, planetree, or mahogany; its face must be planed smooth and flat, and the sides and ends as nearly as possible at right angles to each other — the bottom of the board and the left side should be made perfectly so; and this corner should be marked, so that the stock of the square may be always applied to the bottom and left hand side of the board. To prevent the board from casting, it is usual to pannel it on the back or on the sides.

53. A T square must also be provided, such that by means of a thumb screw fixed in the stock, it may be made to answer either the purposes of a common square, or bevel,—the one-half of the stock being moveable about the screw, and the other fixed at right angles to the blade. The blade ought to be somewhat flexible, and equal in length to the length of the board.

54. Besides these, there will be required a case of mathematical instruments; in the selection of which, it should be observed, that the bow compass is more frequently defective than any of the other instruments. After using any of the ink feet, they should be dried; and if they do not draw properly, they ought to be sharpened and brought to an equal length in the blade, by grinding on a hone.

55. The colours most useful are, Indian ink, gambouge, Prussian blue, vermilion, and lake. With these, all colours necessary for drawing machinery or buildings may be made; so that, instead of purchasing a box of colours, we would advise that those for whom this book is intended should procure these cakes separately,—the gambouge may be bought from an apothecary—a penny-worth will serve a life-time. In choosing the rest, they should be rubbed against the teeth, and those which feel smoothest are of the best quality.

56. Hair pencils will also be necessary, made of camel's hair, and of various sizes. They ought to taper gradually to a point, when wet in the mouth, and should, after being pressed against the finger, spring back.

57. Black-lead pencils will also be necessary. They ought not to be very soft, nor so hard that their traces cannot be easily erased by the India rubber. In choosing paper, that which will best suit the kind of drawing is thick, and has a hardish feel, not very smooth on the surface, yet free from knots.

58. The paper on which the drawing is to be made, must be

chosen of a good quality and convenient size. It is then to be wet with a sponge and clean water, on the opposite side from that on which the drawing is to be made. When the paper absorbs the water, which may be seen by the wetted side becoming dim, as its surface is viewed slantwise against the light. It is to be laid on the drawing board with the wetted side next the board. About half an inch must be turned up on a straight edge all round the paper, and then fastened on the board. This is done because the paper when wet is enlarged, and the edges being fixed on the board act as stretchers when the paper contracts by drying. To prevent the paper from contracting before the paste has been sufficiently fastened by drying, the paper is usually wet on the upper surface, to within half an inch of the paste mark. When the paper is thoroughly dried, it will be found to lie firmly and equally on the board, and is then fit for use.

59. If the drawing is to be made from a copy, we ought first to consider what scale it is to be drawn to. If it is to be equal in size to, or larger than the copy; and a scale should be made accordingly, by which the dimensions of the several parts of the drawing are to be regulated. The diagonal scale, a simple and beautiful contrivance, will be here found of great use for the more minute divisions; and whenever the drawing is to be made to a scale of 1 inch, $\frac{1}{2}$ inch, $\frac{1}{4}$ inch to the foot, a scale should be drawn of 20 or 30 equal parts; the last of which should be subdivided into 12, and a diagonal scale formed on the same principles as the common one, but with eight parallels and 12 diagonals, to express inches and eighths of an inch. For making such scales to any proportion, the line L on the sector will be found very convenient.

60. Great care should be taken in the penciling, that an accurate outline be drawn, for on this much of the value of the picture will depend. The pencil marks should

be distinct, yet not heavy, and the use of the rubber should be avoided as much as possible, as its frequent application ruffles the surface of the paper. The methods already given for constructing geometrical figures will be here found applicable, and the use of the T square, parallel ruler, &c., will suggest themselves whenever they require to be employed.

61. The drawing thus made of any machine or building is called a plan. Plans are of three kinds—a ground plan, or bird's eye view, an elevation or front view, and a perspective plan. This last, however, is seldom employed, and we shall pass it over.

62. When a view is taken of the teeth of a wheel, with the circumference towards the eye, the teeth appear to be nearer as they are removed from the middle point of the circumference opposite the eye, and it may not be out of place here to give the method of representing them on paper:—If AB (fig. 73.) be the circumference of a wheel as viewed by the eye, and it is required to represent the teeth as they appear on it. Only half of the circumference can be seen in this way at one time, consequently we can only represent the half of the teeth. On AB describe a semicircle, which divide into half as many equal parts as the wheel has teeth; then from each of these points of division draw perpendiculars to the wheel AB, then will these perpendiculars mark the relative places of the teeth.

64. When the outline is completed in pencil, it is next to be carefully gone over with Indian ink, which is to be rubbed down with a little water, on a plate of glass or earthenware—so as to be sufficiently fluid to flow easily out of the pen, and at the same time have a sufficient body of colour. While drawing the ink lines the measurement should all be repeated, so as to correct any error that may have slipped during the penciling. The screw in the drawing pen will regulate the breadth of the strokes; which should not be alike heavy; these strokes being the heaviest which

bound the dark part of the shades. Should any stroke chance to be wrong drawn with the ink, it may be taken out by means of a sponge and water, which could not be done if common writing ink were employed.

65. In preparing for colouring it is to be observed, that a hair pencil is to be fixed at each end of a small piece of wood, made in the form of a common pencil, one of which is to be used with colour, and the other with water only. If the colour is to be laid on, so as to represent a flat surface, it ought to be spread on equally, and there is here no use for the water brush; but if it is to represent a curved surface, then the colour is to be laid on the part intended to be shaded, and softened towards the light by washing with the water brush. In all cases it should be borne in mind, that the colour ought to be laid on very thin, otherwise it will be more difficult to manage, and will never make so fine a drawing.

66. In colours even of the best quality, we sometimes meet with gritty particles, which it is desirable to avoid. Instead of rubbing the colour on a plate with a little water as is usual, it will be better to wet the colour, and rub it on the point of the forefinger, letting the dissolved part drop off the finger on to the plate.

67. In using the Indian ink it will be found advantageous to mix it with a little blue and a small quantity of lake, which renders it much more easily wrought with, and this is the more desirable as it is the most frequently used of all the other colours in Mechanical Drawing, the shades being all made with this colour.

The depth and extent of the shades will depend on various circumstances—on the figure of the object to be shaded, the position of the eye of the observer, and the direction in which the light comes, &c. The position of the eye will vary the proportionate size of any object in a picture when drawn in perspective. Thus, if a perspective view of a steam engine

is given, the eye being supposed to be placed opposite the end nearest the nozzles, an inch of the nozzle rod will appear much larger than an inch of the pump rod which feeds the cistern; but if the eye is supposed to be placed opposite the other end of the engine, the reverse will be the case. But in drawing elevations and ground plans of machinery, every part of the machine is drawn to the proper scale—an inch or foot in one part of the machine, being just the same size as an inch or foot in any other part of the machine. So that by measuring the dimensions of any part of the drawing, and then applying the compass to the scale, we determine the real size of the part so measured. Whereas, if the view were given in perspective, we would be obliged to make allowance for the effect of distance, &c. &c.

68. The light is always supposed to fall on the picture at an angle of forty-five degrees, from which it follows, that the shade of any object, which is intended to rise from the plane of the picture, or appear prominent, will just be equal in length to the prominence of the object. Thus a post standing out from a wall, will be represented as in fig. 74. and another standing out twice as far will be represented as in fig. 75. In this way it will be seen that an elevation gives the length, breadth, and thickness of any object, and so also the ground plan.

69. The shades therefore should be as exactly measured as any other part of the drawing, and care should be taken that they all fall in the proper direction, as the light is supposed to come from one point only.

70. It is frequently of great use for the mechanic to take a hasty copy of a drawing, and many methods have been given for this purpose—by machines, tracing, &c. We give the following as easy, accurate, and convenient.

Mix equal parts of turpentine and drying oil, and with a rag lay it on a sheet of good silk paper, allowing the paper to lie by for two or three days to dry, and when it is so it

will be fit for use. To use it, lay it on the drawing to be copied, and the prepared paper being nearly transparent, the lines of the drawing will be seen through it, and may be easily traced with a black lead pencil. The lines on the oiled paper will be quite distinct when it is laid on white paper. Thus, if the mechanic has little time to spare, he may take a copy and lay it past to be recopied at his leisure.

Care and perseverance are the chief requisites for attaining perfection in this species of drawing. Every mechanic should know something of it, so that he may the better understand how to execute plans that may be submitted to him, or make intelligible to others any invention he himself may make. It might have been thought that we would have said more on the depth and extent of the shades of curved surfaces, but the limits of this work would not permit,—nature here should be consulted, and by frequently examining the appearance of machines actually constructed, a just idea may be formed of how they ought to be represented in a plan.

CONIC SECTIONS.

DEFINITIONS.

Conic Sections are the figures made by a plane cutting a cone.

According to the different positions of the cutting plane there arise five different figures or sections, namely, a triangle, a circle, an ellipsis, an hyperbola, and a parabola: the three last of which only are peculiarly called Conic Sections.

If the cutting plane pass through the vertex of the cone, and any part of the base, the section will evidently be a triangle; as VAB.—(Fig. 76.)

If the plane cut the cone parallel to the base, or make no angle with it, the section will be a circle; as ABD.—(Fig. 77.)

The section DAB is an ellipse when the cone is cut obliquely through both sides, or when the plane is inclined to the base in a less angle than the side of the cone is.—(Fig. 78.)

The section is an parabola, when the cone is cut by a plane parallel to the side, or when the cutting plane and the side of the cone make equal angles with the base.—(Fig. 79.)

The section is an hyperbola, when the cutting plane makes a greater angle with the base than the side of the cone makes.

And if all the sides of the cone be continued through the vertex, forming an opposite equal cone, and the plane be also continued to cut the opposite cone, this latter section

will be the opposite hyperbola to the former; as *dBe*.—(Fig. 80.)

The Vertices of any section, are the points where the cutting plane meets the opposite sides of the cone, or the sides of the vertical triangular section; as A and B.

Hence the ellipse and the opposite hyperbolas, have each two vertices; but the parabola only one; unless we consider the other as at an infinite distance.

The Axis, or Transverse Diameter, of a conic section, is the line or distance AB between the vertices.

Hence the axis of a parabola is infinite in length, A*b* being only a part of it.—(Fig. 81.)

The centre C is the middle of the axis.

Hence the centre of a parabola is infinitely distant from the vertex. And of an ellipse, the axis and centre lie within the curve; but of an hyperbola, without.

A Diameter is any right line, as AB or DE, drawn through the centre, and terminated on each side by the curve; and the extremities of the diameter, or its intersections with the curve, are its vertices.

Hence all the diameters of a parabola are parallel to the axis, and infinite in length. Hence also every diameter of the ellipse and hyperbola has two vertices; but of the parabola, only one; unless we consider the other as at an infinite distance.

The Conjugate to any diameter, is the line drawn through the centre, and parallel to the tangent of the curve at the vertex of the diameter. So, FG, parallel to the tangent at D, is the conjugate to DE; and HI, parallel to the tangent at A, is the conjugate to AB.

Hence the conjugate HI, of the axis AB, is perpendicular to it.

An Ordinate to any diameter, is a line parallel to its conjugate, or to the tangent at its vertex, and terminated by

the diameter and curve. So DK, EL, are ordinates to the axis AB; and MN, NO, ordinates to the diameter DE.

Hence the ordinates of the axis are perpendicular to it.

An Absciss is a part of any diameter contained between its vertex and an ordinate to it; as AK or BK, or DN or EN.

Hence, in the ellipse and hyperbola, every ordinate has two determinate abscisses; but in the parabola only one; the other vertex of the diameter being infinitely distant.

The Parameter of any diameter is a third proportional to that diameter and its conjugate, in the ellipse and hyperbola, and to one absciss and its ordinate in the parabola.

The Focus is the point in the axis where the ordinate is equal to half the parameter. As K and L, where DK or EL is equal to the semi-parameter. The name focus being given to this point from the peculiar property of it mentioned in the corol. to theor. 9 in the Ellipse and Hyperbola following, and to theor. 6 in the Parabola.

Hence the ellipse and hyperbola have each two foci; but the parabola only one.—(Fig. 82.)

If DAE, FBG, be two opposite hyperbolas, having AB for their first or transverse axis, and *ab* for their second or conjugate axis. And if *dae*, *fbg*, be two other opposite hyperbolas having the same axes, but in the contrary order, namely, *ab* their first axis, and AB their second; then these two latter curves *dae*, *fbg*, are called the conjugate hyperbolas to the two former DAE, FBG; and each pair of opposite curves mutually conjugate to the other; being all for convenience of investigation referred to one plane, though they are only posited two and two in one plane; as will appear more evidently from the demonstration of th. 2 Hyperbola.

And if tangents be drawn to the four vertices of the curves, or extremities of the axes, forming the inscribed rectangle HIKL; the diagonals, HCK, ICL, of this

rectangle, are called the asymptotes of the curves. And if these asymptotes intersect at right angles, or the inscribed rectangle be a square, or the two axes AB and *ab* be equal, then the hyperbolas are said to be right-angled, or equilateral.

PROBLEMS FOR THE CONIC SECTIONS.

THE PARABOLA.

1. Given two abscissas A and B together with the ordinate of A, to find the ordinate of B.

$$\frac{\text{abscissa B} \times \text{ordinate A}}{\text{abscissa A}} = \text{ordinate B.}$$

Ex.—An abscissa is 9, and its ordinate is 16, it is required to find the ordinate of another abscissa 16.

$$\frac{36^{\frac{1}{2}} \times 16}{9^{\frac{1}{2}}} = \frac{6 \times 16}{3} = 32 \text{ the required ordinate.}$$

2. Given the ordinate and abscissa, required the parameter.

$$\frac{\text{ordinate}^2}{\text{abscissa}} = \text{parameter.}$$

Ex.—The ordinate being 12 and abscissa 6, then,

$$\frac{12^2}{6} = \frac{144}{6} = 24 = \text{the parameter required.}$$

3. To find the length of the curve of a parabola, cut off by a double ordinate to the axis.

$(\text{ordin.}^2 \times \frac{4}{3}\,\text{abs.}^2)^{\frac{1}{2}} \times 2 =$ the length of the curve.

Ex.—The length of the double ordinate being 12 and the abscissa 2, then,

$(6^2 \times \frac{4}{3} \times 2^2)^{\frac{1}{2}} \times 2 = 12{\cdot}858 =$ the length of curve.

NOTE.—This rule is sufficiently correct for practice, but will not apply when the abscissa is greater than the half ordinate.

THE ELLIPSE.

1. To find an ordinate we have the proportion.

As the transverse axis is to the conjugate, so is the square root of the product of the two abscissas, to the ordinate.

Ex. The transverse axis being 60, the conjugate 45, the one abscissa 12, and the other 48, then,

$60 : 45 :: (48 \times 12)^{\frac{1}{2}} : 18 =$ the ordinate required.

2. To find the abscissa.

$$\frac{(\text{the half conju.}^2 - \text{ordin.}^2)^{\frac{1}{2}} \times \text{trans. axis}}{\text{conjugate axis}} =$$

distance between the ordinate and centre of the axis,

which being added to the half axis, will give the greater abscissa, or being subtracted will give the shorter abscissa.

Ex.—One axis being 20 and the other 15, what are the abscissas to the ordinate whose length is 6.

$\frac{(7{\cdot}5^2 - 6^2)^{\frac{1}{2}} \times 20}{15} = 6 =$ the distance from the centre, wherefore $10 + 6 = 16 =$ the longer abscissa, and $10 - 6 = 4 =$ the shorter.

3. To find the conjugate axis.

As (one abscissa $\times$ other abscissa)$^{\frac{1}{2}}$ is to their ordinate, so is the transverse axis to the conjugate.

Ex.—The transverse axis being 200 the ordinate 60, one abscissa is 40 and the other 160, therefore,

$(160 \times 40)^{\frac{1}{2}} : 60 :: 200 : 150 =$ the conjugate axis.

4. To find the transverse axis.

Take the square root of the difference of the squares of the ordinate and half conjugate, and add to this the half conjugate if the lesser abscissa is used, but subtract the half conjugate if the greater abscissa is used. In either case call the result of this part of the operation M, then,

$$\frac{\text{conjugate} \times \text{abscissa} \times \text{M}}{\text{ordinate}^2} = \text{transverse axis.}$$

Ex.—If the ordinate be 20, the lesser abscissa 14, and the conjugate 50, then by the above,

$$(25^2 - 20^2)^{\frac{1}{2}} + 25 = 40 = \text{M}.$$

$$\frac{50 \times 14 \times 40}{20} = 70 = \text{the transverse axis.}$$

5. To find the circumference of an ellipse.

$$\left(\frac{\text{sum of the sq. of the two axes}}{2}\right)^{\frac{1}{2}} \times 3{\cdot}1416 = \text{circumference.}$$

Ex.—The one axis being 24 and the other 18, then,

$$\left(\frac{24^2 + 18^2}{2}\right)^{\frac{1}{2}} \times 3{\cdot}1416 = 66{\cdot}643 = \text{circumference.}$$

THE HYPERBOLA.

1. *To find the ordinate.*

As the transverse axis is to the conjugate; so is the square root of the product of the two abscissas, to the ordinate.

Ex.—The transverse axis being 24, the conjugate 21, and the abscissa 8; then,

$$24 : 21 :: (32 \times 8)^{\frac{1}{2}} : 14 = \text{the ordinate.}$$

2. *To find the abscissas.*

$$\frac{(\text{ord.}^2 + \text{half conju.}^2)^{\frac{1}{2}} \times \text{trans. axis}}{\text{conjugate}} = \text{dist. between the ordin. and centre.}$$

Then this distance, added to the half transverse, gives the greater abscissa; or, subtracted from it, the less.

Ex.—The transverse axis being 40, the conjugate 32, and the ordinate 12; then,

$$\frac{(16^2 + 12^2)^{\frac{1}{2}} \times 40}{32} = 25 = \text{distance from the middle of the transverse.}$$

Wherefore, $25 + 20 = 45 =$ the greater abscissa; and $25 - 20 = 5 =$ the lesser.

3. *To find the conjugate.*

$$\frac{\text{ordinate} \times \text{transverse axis}}{(\text{product of the abscissas})^{\frac{1}{2}}} = \text{conjugate.}$$

Ex.—The transverse axis being 144, the lesser abscissa 48, and its ordinate 84; then,

$$\frac{84 \times 144}{(192 \times 48)^{\frac{1}{2}}} = 126 = \text{the conjugate required.}$$

4. *To find the transverse.*

According as the lesser or greater abscissa is used, take the sum or difference of the half conjugate, and the sum of the squares of the half conjugate and ordinate, and call this result m; then,

$$\frac{\text{abscissa} \times \text{conjugate} \times m}{\text{ordinate}^2} = \text{the transverse axis.}$$

Ex.—The conjugate being 18, the lesser abscissa 10, and its ordinate 12; then,

$$(12^2 + 9^2)^{\frac{1}{2}} + 9 = 15 + 9 = 24 = m;$$

$$\frac{10 \times 18 \times 24}{144} = 30 = \text{the transverse axis.}$$

USEFUL CURVES.

The Cycloid is a very useful curve; and may be defined, the curve formed by a nail in the rim of a wheel, while it moves along a level road. The cycloid may be described on paper, thus:—Cut a circle of stiff pasteboard, and having placed a flat ruler on the paper, roll the circle along its edge, keeping a pencil at one point of the circumference; then, by the motion of the circle, the pencil will describe a cycloid. By an examination of fig. 83, this will be easily understood.

If the ball of a pendulum be made to move in a cycloid,

its vibrations will be isocronous, or they will all be performed in the same time. The cycloid is also the line of swiftest descent; or a body will fall through the arc of this curve, from one given point to another, in less time than through any other path.

The Catenary is that curve which is formed by a chain or chord of uniform texture, when it is hung upon two points, and left to hang freely, without any restraint. It matters not whether these points of suspension be in the same horizontal line or not, or whether the chain be slack or tight; still the curve will be a catenary.—A knowledge of this curve is very useful in the construction of suspension bridges.

MENSURATION.

DEFINITIONS.

1. A *plane* is a solid, of which the sides are parallelograms, and the ends equal, similar, and parallel plane figures. The figure of the ends gives the name to the prism; if the ends are triangular, the prism is triangular, &c. If the sides and ends of a prism be all equal squares, the prism is called a cube; and if the base or ends be a parallelogram, the prism is called a Parallelopipedal. The cylinder is a round prism, having circular ends.

2. The *pyramid* has any plane figure for its base and its sides triangles, of which all the Vertices meet in a point at the top, called the Vertex of the pyramid.

3. A *sphere* or *globe* is a solid bounded by one continued surface, every point of which surface is equally distant from a point within the sphere, called the Centre. The Diameter

or Axis of a sphere, is any line which passes through its centre, and is terminated at both ends by the circumference.

4. A *prismoid* has its two ends as any unlike parallel plane figures of the same number of sides; the upright sides being Trapezoids.

5. A *spheroid* is a solid resembling the figure of a sphere, but not exactly round—one of its diameters being longer than the other; and, likewise, a Conoid is like a cone, but has not its sides straight lines but curved.

6. A *spindle* is a solid formed by the revolution of some curve round its base.

7. The *axis* of a solid is a straight line drawn through the solid, from the middle of one end to the middle of the opposite end.

8. The *height* of a solid is a line drawn from the vertex perpendicular to the base, or the plane on which the base rests.

9. The *segment* of a solid is a part cut off by a plane, parallel to the base; and the *frustum* is the part remaining after the segment is cut off.

10. A *polyedrum* is a solid bounded by planes.

SURFACES.

1. *For the area of a square, rhombus, or rhomboid.*

$$\text{Base} \times \text{height} = \text{area.}$$

Ex.—The basis of a rhombus is 16, the height 9; therefore, $16 \times 9 = 144 =$ area.

2. *For the area of a triangle.*

$$\tfrac{1}{2}(\text{base} \times \text{height}) = \text{area.}$$

Ex.—The base of a triangle is $2\frac{1}{4}$, and height $7\frac{1}{2}$; therefore, $\frac{1}{2}(2{\cdot}25 \times 7{\cdot}5) = 8{\cdot}387$, the area.

3. *For the area of a trapezoid.*

$\frac{1}{2}$ (sum of the two parallel sides) $\times$ height $=$ area.

Ex.—In a trapezoid one of the parallel sides is $16\frac{1}{8}$, the other is $14\frac{1}{4}$, and the height or perpendicular distance between them is 7; therefore,

$$\tfrac{1}{2}(16{\cdot}125 + 14{\cdot}25) \times 7 = 106{\cdot}309, \text{ the area.}$$

4. *For any right-lined figure of four or more unequal sides.*

Divide it into triangles, by lines drawn from various angles; find the area of each; then, the sum of these areas will be the area of the whole figure.

5. *For a regular polygon.*

Inscribe a circle; then, $\frac{1}{2}$ (radius of insc. circle $\times$ length of one side $\times$ number of sides) $=$ area.

Ex.—In a polygon of 8 sides, the length of a side is 16, and radius of inscribed circle 3;

then, $\frac{1}{2}(3 \times 16 \times 8) = 192$, the area. Otherwise, multiply the square of one side of the polygon, by that number which stands against the number of the sides of the figure, in this table; the product is the area.

5.	1·72047,	9.	6·18182
6.	2·59807,	10.	7·69421
7.	3·6339,	11.	8·51425
8.	4·8284,	12.	9.33012

Thus, in a figure of 10 equal sides, one side is 8, and $8^2 = 64$; hence, $64 \times 7{\cdot}69421 = 482{\cdot}42844$, the area.

6. *For the circle.*

1*st*, diameter $\times$ 3·1416 $=$ circumference;

2*nd*, $\dfrac{\text{circumference}}{3{\cdot}1416} =$ diameter;

3*rd*, $\frac{1}{2}$ circumference $\times$ radius $=$ area.

Ex.—In a circle whose diameter is 14 inches, we have,

1*st*, $14 \times 3{\cdot}1416 = 43{\cdot}9804$, the circumference;

2*nd*, $\dfrac{43{\cdot}9804}{3{\cdot}1416} = 14$, the diameter;

3rd, diameter 14 = 2 radius; therefore,

$\frac{1}{2}$ (43·9804) × 7 = 153·3307, the area.

7. *For the length of the arc of a circle.*

Radius × ·079577 × number of degrees = length of ar

Ex.—If the radius be 12, and number of degrees 24, the

12 × ·079577 × 22 = 21·008328, the length.

8. *For the area of a circular sector.*

Radius × $\frac{1}{2}$ length of arc.

Ex.—The radius being 12, and length of arc 21·00832 then, 12 × 10·514164 = 126·049968, the area.

9. *For the area of a circular segment.*

Find the area of a sector of the same radius, and lengt of arc, with the segment; then find the area of the triang formed by the two radii and the chord of the arc; the the difference, or sum of these areas, will be that of the se ment, according as it is greater or less than a semicircle.

Ex.—In the last example, the area of the sector w 126·049968, 27·3717 = area of the segment; this 27·371 being the area of the triangle formed by the chord an radii.

10. *For the area of a cycloid.*

Area of generating circle × 3 = area of cycloid.

Ex.—The diameter of generating circle being 10, then $\frac{1}{2}$ (10 × 3·1416) × 3 = 235·619, the area of cycloid.

11. *For the area of a parabola.*

(Base × height) × $\frac{2}{3}$ = the area.

Ex.—The base being 20, and height 6; then,

20 × 6 × $\frac{2}{3}$ = 51·423, the area.

12. *For the area of an ellipse.*

(Long axis × short axis) × ·7854 = area.

Ex.—The greater axis being 300, and lesser 200; then,
$300 \times 200 \times \cdot 7854 = 47124$, the area.

13. *For the surface and content of a prism.*

Area of two ends + length × perimeter, equal surface.

A triangular prism is 30 long, and the ends of the base each 31.

The area of the base is $3\cdot798$; hence $9 \times 30 + 7\cdot596 = 277\cdot586 =$ surface.

Content = area of base × height;

Therefore $3\cdot798 \times 30 = 11\cdot3940 =$ content.

The same rules hold for the cylinder.

14. *For a cone or pyramid.*

$\frac{1}{2}$(slant height × perimeter of base) + area of base = surface.

$\frac{1}{3}$ (area of base × perpendicular height) = content.

Ex. Slant height is 10, perimeter of base 16; then, $\frac{1}{2}(10 \times 16) = 80 + 16 = 96$, surface of a four-sided pyramid, whose side at the base is 4.

The area of the base of a cone being $147\cdot68$, and perpendicular height 14,

Then $\frac{1}{3}(14 \times 147\cdot68) = 675\cdot86$, content.

15. *For a cube or paralelopiped.*

The sum of the areas of all the sides = surface.
length × breadth × depth = content.

Ex. In a paralelopiped the length 30, breadth 6, and depth 4.

$30 \times 6 \times 4 = 720 =$ content, and $648 =$ the surface.

16. *For regular or platonic bodies, or bodies of equal sides.*

Linear edge $^2 \times$ tabular number of figures for surface = surface.

Linear edge $^3 \times$ tabular number of figures for solidity = content.

No. of Sides.	Name.	Multiplier for Surface.	Multiplier for Solidity.
4	Tetrahedron,	1·7320508	0·1178513
6	Hexaedron,	6·0000000	1·00000
8	Octaedron,	3·4641016	0·4714045
12	Dodacaedron,	20·6457288	7·6631189
20	Icosaedron,	8·6602540	2·181695

Ex.—In an Octaedron the length of the ridge of a side is 5, therefore $5^2 \times \cdot 3 \cdot 4641016 = 86{\cdot}6025$ = surface, and $5^2 \times \cdot 4714045 = 58{\cdot}9255$, the solidity.

17. *For the surface or the segment of a sphere.*

Diameter $^2 \times 3{\cdot}1416$ = surface of the whole sphere.

Ex.—If the diameter be 36, then $36^2 \times 3{\cdot}1416 = 4071{\cdot}504$ square inches = surface.

Height of segment $\times$ diameter of sphere $\times$ $3{\cdot}1416$ = surface of segment.

Ex.—The diameter of the sphere being 12, and the height of segment 6, then
$6 \times 12 \times 3{\cdot}1416 = 226{\cdot}1856$ = surface of spheric segment.

18. *For the content of a sphere and spheric segment.*

Diameter $^3 \times 0{\cdot}5236$ = content.

Ex.—If the diameter of a sphere be 36 inches, then $36^3 \times 0{\cdot}5236 = 678{\cdot}584$ = the content.

(radius of segment, base $^2 \times 3 +$ height of segment^2) $\times$ height $\times$ $\cdot5236$ = content of segment.

Ex.—If the height of a spheric segment be 2, and radius of base 6, then

$$(6^2 \times 3 \times 2^2) \times 2 \times \cdot 5236 = 117{\cdot}286 = \text{content.}$$

19. *For the solidity of a spheroid.*

Revolving axis $^2 \times$ fixed axis $\times$ $\cdot5236$ = content.

Note.—If the spheroid revolve round the greater axis, it is said to be oblate; if round the lesser, oblong.

Ex.—The two axes of a spheroid are 24 and 18; therefore,

$24^2 \times 18 \times \cdot 5236 = 5428\cdot 6 =$ content if it be oblate.

$18^2 \times 24 \times \cdot 5236 = 4071\cdot 5 =$ content if it be oblong.

20. *For the solidity of a parabolic conoid.*

Area of base $\times$ half the height $=$ content.

Ex.—The height being 18, and the diameter of base 24, then the area of the base therefore is $452\cdot 39$; hence

$452\cdot 39 \times 9 = 4071\cdot 51$ the content.

21. *For the frustum of a cone or pyramid.*

$$\frac{(\text{perim. of one end} \times \text{perim. of the other end}) \times \text{slant height}}{2} = \text{surface.}$$

Ex.—In the frustum of a triangular pyramid the perimeter of one end is 25, that of the other 36, and the slant height is 10; therefore,

$$\frac{(25 + 36) \times 10}{2} = 305 = \text{the surface.}$$

$$\frac{(\text{area of one end} + \text{ar. of other})^{\frac{1}{2}} + \text{area of one end} + \text{ar. of other}}{3} \times \text{height} = \text{content}$$

Ex.—A log of wood is 20 feet long; its ends are squares, of which the sides are respectively 12 and 16 inches; therefore,

$$\frac{(12^2 + 16^2)^{\frac{1}{2}} + 12^2 + 16^2}{3} \times 240 = 3725\cdot 6 \text{ inches} = 27\cdot 4 \text{ feet,}$$

the content of the log.

TIMBER MEASURE.

NO ALLOWANCE FOR BARK.

LENGTH OF TREE AND GIRTH, BOTH TAKEN IN INCHES.

$$\frac{\text{length} \times \text{girth}^2}{2304} = \text{cubic feet customary.}$$

$$\frac{\text{length} \times \text{girth}^2}{1807} = \text{cubic feet, true content.}$$

ALLOWING ONE-EIGHTH FOR BARK.

$$\frac{\text{length} \times \text{girth}^2}{3009} = \text{cubic feet customary.}$$

$$\frac{\text{length} \times \text{girth}^2}{2360} = \text{cubic feet, true content.}$$

Ex.—The tree being 40 long and 5 feet in girth, or circumference, what is its content, no allowance being made for bark?

40 feet = 480 inches, and 5 feet = 60 inches; therefore,

$$\frac{480 \times 60^2}{2304} = 62\tfrac{1}{2} \text{ cubic feet customary.}$$

$$\frac{480 \times 60^2}{1807} = 79\tfrac{3}{4} \text{ cubic feet, true content.}$$

A tree 50 feet long and 4 feet 1 inch girth, allowance being made for bark.

50 feet = 600 inches, and 4 feet 1 inch = 49 inches.

$$\frac{600 \times 49^2}{3009} = 40 \text{ cubic feet customary.}$$

$$\frac{600 \times 49^2}{2360} = 50\tfrac{3}{4} \text{ cubic feet, true content.}$$

Note.—Trees very seldom have an equal girth throughout, one end being generally much smaller than the other: the girth taken above is the mean girth; that is to say, the girths of both ends added together, and their sum halfed for the mean girth. It is to be observed, however, that, if the difference of the girths is great, it will be best to find the content of the tree as if it were a conic frustum.—The method of using the sliding-rule in the measurement of timber, has been given before.

HEIGHTS AND DISTANCES.

As in this book we have not given either tables or instructions for Trigonometric calculation, we have thought it

necessary to give the following simple method of measuring heights and distances, which will be found sufficiently exact for the purposes of those for whom these pages are designed.

AB, fig. 84, is a staff five feet long, with the end A pointed so as to stick firmly in the ground. On the top B there is cut a groove, such, that it will exactly receive the square rod CD, at the end of which rod, at C, there is fastened a small piece of brass plate, with a hole in it to look through, which is called a sight. At the other end of the rod there is placed a slip of brass, which slides up and down in the rod, and may be fixed at any point by means of a small thumb-screw at D. The distance of this slip from the sight C is exactly one foot. At the top E of this slip there is a hole, over which a hair is fastened, and from this hair the slip is divided into equal parts, each being exactly the tenth part of a foot, and each of these divisions is again divided into ten smaller divisions, each of which smaller divisions will therefore be equal to the hundredth part of a foot; so that, if one foot be reckoned unity, the smaller divisions will be decimal parts of tenths and hundredths. Thus, 2 of the larger divisions would be ·2, but 2 of the smaller would be ·02; and 14 of the larger, and 6 of the smaller, would be 1·46. There is also placed a pointed pin in the rod CD, just at the bottom of the brass slip; and as long as the hole in the sight C is above the level of the rod, the use of this pin is to mark the divisions to be counted on the brass slip.

To measure the height of a steeple with this instrument, proceed thus:—Set its pointed end in the ground, and adjust it so that it will stand firm, and the rod CD be level, and directed towards the object whose height is to be measured; then, looking through the sight C to the top of the steeple, shift the brass slider up and down till the hair over the hole at its top appears to be in the same line with the top of the steeple. Fasten the slip by the thumb-screw, and see at what division the point of the pin stands, and mark

them. Then measure the distance in feet betwixt the point where the instrument stands and the bottom of the steeple, then this distance, multiplied by the divisions before marked from the slip, will give the height of the steeple.—Thus, if the instrument be placed at 120 feet distant from Nelson's monument in Glasgow green, and, after viewing its top as above directed, the pin-marks 1·1, that is, one of the larger divisions and one of the smaller, then $1·1 \times 120 = 132$, the height of the monument. It is to be remembered, however, that the height of the sight C above the ground must be added to the height of the object, as given by the above process. If the height of the sight C be 5 feet, then the height of the monument would be $132 + 5 = 137$ feet.

The same instrument may be used for measuring horizontal distances, if it is so formed that the brass slip can be made to move horizontally. The particulars of this the reader will easily perceive, so that it is unnecessary for us to say more on the subject.

MECHANICS.

DEFINITIONS.

1. A BODY is any quantity of matter collected together.

2. Whatever communicates, or has a tendency to communicate, motion to a body, is called a force.

3. That department of knowledge which comprehends a statement of the effects of forces on bodies, is called Mechanics. If a body be put in motion by the action of one or more forces, the consideration of the circumstances of this body belongs to that branch of Mechanics called Dynamics; but if two or more forces act on a body in such a way that they destroy each other's effects, and the body remains at rest, or in equilibrium, the consideration of the circumstances of a body, in this case, belongs to that department of Mechanics called Statics.

4. The density of matter, is the quantity of matter contained in any body compared with its bulk. Thus lead is denser than cork.

5. The weight of a body, is its quantity of matter, without regard to its bulk.

6. When we speak of some given space, which a moving body passes over in a given time, we speak of the velocity of the body. If a body moves over one foot of space in one second of time, it is said to have a velocity of one foot in the second; and its velocity would be increased to the double, if it passed over two feet in one second of time.

7. If, while the body is in motion, the velocity continues the same, the body is said to have a *uniform* motion; but if,

while the body moves onward, the velocity continually increases, it is said to have an *accelerated* motion; and, on the other hand, if during the progress of the body in motion, the velocity continually decreases, the body is said to have a *retarded* motion.

8. The quantity of matter in a moving body, multiplied by the velocity with which it moves, is called the *quantity of motion*, or *momentum* of the body.

9. Gravity is that force by which all bodies endeavour to descend towards the centre of the earth.

AXIOMS, OR PLAIN TRUTHS.

If a body be at rest, it will remain at rest; and if in motion, it will continue that motion, uniformly in a straight line, if it be not disturbed by the action of some external cause.

The change of motion takes place in the direction in which the moving force acts, and is proportional to it.

The action and reaction of bodies upon one another, are equal.

LAWS OF MOTION.

Uniform motion is caused by the action of some force, by one impulse, on the body; and if

b signify the quantity of matter to be moved,
f the force which caused the body's motion,
v the velocity with which the body moves,
m the momentum of the body in motion,
s the space passed over by the moving body,
t the time of describing that space;

and if $b = 3$, $m = 6$, $v = 2$, $f = 6$, $s = 4$, and $t = 2$: then the figures in the examples will show the application of the theorems.

THEOREMS.	EXAMPLES.
$b : \frac{m}{v} : \frac{f}{v} : \frac{m \times t}{s} : \frac{f \times t}{s}$	$3 : \frac{6}{2} : \frac{6}{2} : \frac{6 \times 2}{4} : \frac{6 \times 2}{4}$
$f : m : b \times v : \frac{b \times s}{t}$	$6 : 6 : 3 \times 2 : \frac{3 \times 4}{2}$
$m : f : b \times v : \frac{b \times s}{t}$	$6 : 6 : 3 \times 2 : \frac{3 \times 4}{2}$
$s : t \times v : \frac{t \times m}{b} : \frac{t \times f}{b}$	$4 : 2 \times 2 : \frac{2 \times 6}{3} : \frac{2 \times 6}{3}$
$v : \frac{m}{b} : \frac{s}{t} : \frac{f}{b}$	$2 : \frac{6}{3} : \frac{4}{2} : \frac{6}{3}$
$t : \frac{s}{v} : \frac{s \times b}{m} : \frac{s \times b}{f}$	$2 : \frac{4}{2} : \frac{4 \times 3}{6} : \frac{4 \times 3}{6}$

OF ACCELERATED MOTION.

If the moving force continues to act all the while that the body is in motion, then that motion will be uniformly accelerated: such is the case with bodies falling to the earth, as the force of gravity acts constantly. Now, it has been found by experiment, that a body falling through free space, in the latitude of London, will, by the force of gravity, fall through 16·095 feet in the first second of time; and as forces are measured by the effects they produce, this 16·095 may be taken as the measure of the force of gravity; and as this quantity does not differ materially from 16 feet, we shall neglect the fraction ·095 in our calculation of the circumstances of falling bodies.

The subjects of consideration here are, the time that the falling body is in motion, the space it falls through in that time, and the velocity which it has acquired in falling through that space, or that velocity with which it would continue to move, supposing gravity to cease its action, and the motion of the body becoming uniform.

The time is always supposed to be taken in seconds, and the space in feet.

The velocity acquired $= 32 \times$ time of falling,

or $= (64 \times \text{space fallen through})^{\frac{1}{2}}$.

The time of falling $= \dfrac{\text{the velocity acquired}}{32}$

or $= \left(\dfrac{\text{the space fallen through}}{16}\right)^{\frac{1}{2}}$

The space fallen through $= \left(\dfrac{\text{the velocity acquired}}{64}\right)^{2}$

or, time$^{2} \times 16$.

Exam.—If a body falls through 100 feet, then

$(64 \times 100)^{\frac{1}{2}} = 80 =$ the velocity acquired,

$\dfrac{80}{32} = 2\dfrac{16}{32} = 2{\cdot}5 =$ the time of falling.

If the space described be mere 64 feet, then

$\dfrac{(64)^{\frac{1}{2}}}{16} = 2 =$ the time,

$32 \times 2 = 64 =$ the velocity acquired.

If the space descended be 400, then

$(400 \times 64)^{\frac{1}{2}} = 160 =$ the velocity acquired,

$\dfrac{(160)^{\frac{1}{2}}}{32} = 5 =$ the time of falling.

If the space be as	1,	2,	3,	4,	5,	&c.
The velocities will be as	1,	2,	3,	4,	5,	&c.
And the spaces as	1,	4,	9,	16,	25,	&c.
The space for each time as	1,	3,	5,	7,	9,	&c.

COLLISION OF BODIES.

If two bodies, A and B, in motion, weigh respectively 5 and 3 lbs., and their velocities respectively 3 and 2 before they strike, then will 3×5 be the momentum of

A, and 2×3 that of B, before the stroke; also, $5 + 3 = 8$ is the sum of their weights; then,

1st. If the bodies move the same way, the quotient arising from the division of the sum of the momentums of the two bodies, by the sum of their weights, will give the common velocity of the two bodies after the stroke.

2nd. If the bodies move contrary ways, then the quotient arising from the division of the difference of their momentums, by the sum of their weights, will give the common velocity after the stroke.

3rd. If one of the bodies be at rest, then the quotient of the momentum of the other body, divided by the sum of the weights of the two bodies, will give the common velocity after the stroke. Hence, assuming the numbers given above, we have, in the first case, $\frac{15 + 6}{8} = 2\frac{5}{8}$, in the second $\frac{15 - 6}{8} = 1\frac{1}{8}$, and in the third $\frac{15}{8} = 1\frac{7}{8}$, as the common velocity after the stroke.

If the bodies were perfectly elastic, the theorems become more complicated.

If the weight of the one body be A, and the velocity V; the weight of the other body B, and its velocity v: then,

1st. If the bodies move in the same direction before the stroke,

$$\frac{(A - B) \times V + 2 \times B \times v}{A + B} = \text{the velocity of A after the stroke.}$$

$$\frac{(A - B) \times v + 2 \times A \times V}{A + B} = \text{the velocity of B after the stroke.}$$

2nd. If B moved in the contrary direction to A before the stroke,

$$\frac{(A - B) \times V - 2 \times B \times V}{A + B} = \text{velocity of A after the stroke.}$$

$$\frac{(A - B) \times v + 2 \times A \times V}{A + B} = \text{velocity of B after the stroke.}$$

3rd. If the body B had been at rest before it was struck by A, then

$$\frac{A - B}{A + B} \times V = \text{the velocity of A after the stroke,}$$

$$\frac{2 \times A}{A + B} \times V = \text{the velocity of B after the stroke.}$$

Exam.—If the weight of an elastic body A be 6 lbs., and its velocity 4, and the weight of another body B be 4 lbs., and its velocity 2; then we have these results: in the first case,

$$\frac{(6-4) \times 4 - 2 \times 4 \times 2}{6 + 4} = {\cdot}8 = \text{velocity of A after the stroke.}$$

$$\frac{(6-4) \times 2 + 2 \times 6 \times 4}{6 + 4} = 5{\cdot}2 \text{ velocity of B after the stroke.}$$

The sum of these two velocities, viz. 5·2 and ·8 = 6, which was the sum of the velocities 2 and 4 before the stroke; and this is a general law.—The reader may exercise himself with the rules for the other cases.

It is to be observed, that when non-elastic bodies, that is, bodies which have no spring, strike, they will both move in the direction of the motion of that body which has the greater momentum; but if they are elastic, they will recoil after the stroke, and move contrary ways.

THE COMPOSITION AND RESOLUTION OF FORCES.

If a body be acted upon by two forces, one of which would cause it to move from A to B (fig. 85,) in any given time, and the other would cause it to move from A to C in the same time; then if these forces act upon the body at one instant, it will move in neither of the lines AB, AC, but in the line AD, which is the diagonal of the parallelogram of which the two lines AB and AC are containing sides; and by the action of the two forces, the body will be found at D, at the end of the time that it would have been found at

B or C, by the action of either of the forces singly. This important fact in mechanical science, is usually called the *parallelogram of forces*. From this statement it will be seen, that if we have the quantity and direction of any two forces urging a body at the same instant, we can find the resulting motion, both in quantity and direction. It will not be difficult to understand, that if the two forces which act upon a body, act not at an angle, but in the same straight line, and in contrary directions, the resulting motion will be in that straight line, and in the direction of the greater force; but if the forces be equal, the body will remain at rest. If, while the body is urged by a force in the direction AB, and another force in AC, and a third force in the direction DA; these being the sides and diagonals of a parallelogram, the body A will remain at rest. Also, if a body A has a tendency to move in the direction AB, but is restrained by a force AD,—and if we wish to keep the body A from moving, altogether, we must apply another force AC, forming the other side of the parallelogram of which AB is one side and AD the diagonal.

If there be three forces acting on a body at the same time, make the sides of a parallelogram represent any two of them; then the diagonal of this parallelogram, together with the third force as the two sides of another parallelogram, will give a diagonal which will be the result of the three forces acting at once on the body.

If the two forces which urge the body, both produce a uniform motion, the resulting motion will be in a straight line; but if one of them act by impulse, which would produce a uniform motion, and the other act constantly so as to produce an accelerated motion, the resulting motion will be in a curve. Thus, if the ball of a cannon were sent in a horizontal direction, it would never deviate from this straight line unless acted on by some external force. The force of gravity acts on the body constantly, so as to draw it to the

earth, by a uniformly accelerated motion; and the result is, that the ball will move in a curve, and this curve may be easily shown to be that of the parabola. The resistance of the air being taken into account together with these circumstances, constitute the bases of the science of gunnery.

We shall give a simple example, to show the application of the former part of this subject. One force will cause the body A to move 20 miles in a day, and another, acting at right angles, will cause it to move 18 miles a day; draw these lines 20 and 18 from the line of lines on the sector, as the sides AB, AC, of a parallelogram, and complete it; draw the diagonal, then measure it, and it will be found to be 26·9, the resulting motion; and the angle being measured, will give the direction.—There are other methods of doing this by calculation, but this is simple, and is sufficient to show the principle.

MECHANIC POWERS.

1. A MACHINE is any instrument employed to regulate motion, so as to save either *time* or *force*. No instrument can be employed by man so as to save both time and force; for it is a maxim in mechanics, that whatever we gain in the one of these two, must be at the expense of the other.

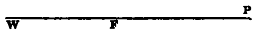
2. The simple machines, or those of which all others are constructed, are usually reckoned six: the *lever*, the *wheel* and *axle*, the *pulley*, the *inclined plane*, the *wedge*, and the *screw*. To these the *funicular machine* is sometimes added.

3. The weight signifies the body to be moved, or the resistance to be overcome; and the power is the force employed to overcome that resistance, or move that body. They are frequently represented by the first letters of their names, W and P.

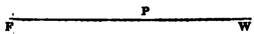
THE LEVER.

4. A lever is an inflexible bar, either straight or bent, supposed capable of turning round a fixed point, called the *fulcrum*.

According to the relative positions of the weight, power, and fulcrum, on the lever, it is said to be of three kinds, viz. when the fulcrum is somewhere betwixt the weight and power, it is of the first kind; when the weight is between the power and the fulcrum, it is of the second kind; and when the power is between the weight and the fulcrum, it is of the third kind: thus,

5. *1st.* 

6. *2nd.* 

7. *3rd.* 

8. In the first and second kinds there is an advantage of power, but a proportionate loss of velocity; and in the third kind, there is an advantage in velocity, but a loss of power.

9. When the weight $\times$ its distance from the fulcrum $=$ the power $\times$ its distance from the fulcrum, then the lever will be at rest, or in equilibrio; but if one of these products be greater than the other, the lever will turn round the fulcrum in the direction of that side whose product is the greater.

10. In all the three kinds of levers, any of these quantities, the weight or its distance from the fulcrum, or, the power or its distance from the fulcrum, may be found from the rest, such, that when applied to the lever, it will remain at rest, or the weight and power will balance each other.

11. $$\frac{\text{weight} \times \text{its dist. from fulc.}}{\text{dist. of power from fulc.}} = \text{power.}$$

12. $$\frac{\text{power} \times \text{its dist. from fulc.}}{\text{dist. of weight from fulc.}} = \text{weight.}$$

13. $\frac{\text{weight} \times \text{dist. weight from fulc.}}{\text{power}} = \text{dist. power from ful.}$

14. $\frac{\text{power} \times \text{dist. power from fulc.}}{\text{weight}} = \text{dist. weight from ful.}$

15. In the first kind of lever, the pressure upon the fulcrum = the sum of weight and power; in the second and third = their difference.

16. If there be several weights on both sides of the fulcrum, they may be reckoned powers on the one side of the fulcrum, and weights on the other. Then, if the sum of the product of all the weights × their distances from the fulcrum be = to the sum of the products of all the powers × their distances from the fulcrum, the lever will be at rest, if not it will turn round the fulcrum in the direction of that side whose products are greatest.

17. In these calculations, the weight of the lever is not taken into account; but if it is, it is just reckoned like any other weight or power acting at the centre of gravity.—This subject will be considered hereafter.

18. When two, three, or more levers act upon each other in succession, then the entire mechanical advantage which they give, is found by taking the product of their separate advantages.

19. It is to be observed, in general, before applying these observations to practice, that if the lever be bent, the distances from the fulcrum must be taken, as perpendiculars drawn from the lines of direction of the weight and power to the fulcrum.

Exam.—In a lever of the first kind, the weight is 16, its distance from the fulcrum 12, and the power is 8; therefore, by No. 13 of this chapter,

$\frac{16 \times 12}{8} = 24 =$ the distance of power from the fulcrum.

In a lever of the second kind, a power of 3 acts at a distance

of 12; what weight can be balanced at a distance of 4 from the fulcrum? Here, by No. 12,

$$\frac{3 \times 12}{4} = 9 = \text{weight.}$$

In a lever of the third kind, the weight is 60, and its distance 12, and the power acts at a distance of 9; therefore, by No. 11,

$$\frac{12 \times 60}{9} = 80, \text{ the power required.}$$

If there be a lever of the first kind, having three weights, 7, 8, and 9, at the respective distances of 6, 15, and 29, from the fulcrum on one side, and a power of 17 at the distance of 9, on the other side of the fulcrum; then a power is to be applied at the distance of 12 from the fulcrum, on the last mentioned side: what must that power be to keep the lever in balance? Here $(6 \times 7) + (15 \times 8) + (29 \times 9) = 403$ = the effect of the weights on one side; and $17 \times 9 = 153$ = the effect of the power on the other side. Now, it is clear that the effect of the weight is far greater than the effect of the power; and the difference $403 - 153 = 250$ requires to be balanced by a power applied at the distance of 12, which will evidently be found by dividing 250 by 12, which gives 20·83, the weight required.

20. Remarks.—The Roman steel-yard is a lever of the first kind, and so contrived, that one weight may be employed to weigh a great many. The fulcrum being placed near one end, say an inch distant, at which end there is hung an ounce weight, then if the other arm be 16 inches long, a weight of one ounce hung at the distance of one inch, will just keep the lever or steel-yard in equilibrium. A weight of two ounces at the distance of two inches will do the same, so three ounces at the distance of three inches, and sixteen ounces, or one lb., at the distance of sixteen inches, &c.; so that, by moving the weight along the steel-yard till we get it to a balance, we may weigh any thing from an ounce

to a pound, by means of an ounce weight at the short end. The divisions may be varied, and also the weight, at the short end, so as to weigh any thing whatever; but the principle remains the same.

The common weighing balance is also a lever of the first kind. The requisites of a good balance are: that the points of suspension of the scales and the centre of motion, or fulcrum of the beam, be all in one straight line—that the arms of the beam be equal to each other in every respect—that they be as long as possible—that the centre of gravity of the beam be a very little below the centre of motion—that the beam be balanced when the scales are empty, &c. But we may ascertain the true weight of any body even by a false balance, thus: weigh the body first in one scale, then in the other, and multiply their weights together; then the square root of this product will be the true weight.

THE WHEEL AND AXLE.

21. The wheel and axle is a kind of lever, so contrived as to have a continued motion about its fulcrum, or centre of motion, where the power acts at the circumference of the wheel, whose radius may be reckoned one arm of the lever, the length of the other arm being the radius of the axle, on which the weight acts. If the power acts at the end of a handspike fixed in the rim of the wheel, then this increases the leverage of the power, by the length of the handspike.—(See fig. 86.)

22. It will be easily seen, that if two wheels fastened together and turning round the same centre, be so adjusted, that while they turn round they will coil on their circumferences strings, to which weights are suspended; one of those wheels being larger than the other, the larger wheel will coil up a greater length of the string than the

of 12; what weight can be balanced at a distance of 4 from the fulcrum? Here, by No. 12,

$$\frac{3 \times 12}{4} = 9 = \text{weight.}$$

In a lever of the third kind, the weight is 60, and its distance 12, and the power acts at a distance of 9; therefore, by No. 11,

$$\frac{12 \times 60}{9} = 80, \text{ the power required.}$$

If there be a lever of the first kind, having three weights, 7, 8, and 9, at the respective distances of 6, 15, and 29, from the fulcrum on one side, and a power of 17 at the distance of 9, on the other side of the fulcrum; then a power is to be applied at the distance of 12 from the fulcrum, on the last mentioned side: what must that power be to keep the lever in balance? Here $(6 \times 7) + (15 \times 8) + (29 \times 9) = 403 =$ the effect of the weights on one side; and $17 \times 9 = 153 =$ the effect of the power on the other side. Now, it is clear that the effect of the weight is far greater than the effect of the power; and the difference $403 - 153 = 250$ requires to be balanced by a power applied at the distance of 12, which will evidently be found by dividing 250 by 12, which gives 20·83, the weight required.

20. Remarks.—The Roman steel-yard is a lever of the first kind, and so contrived, that one weight may be employed to weigh a great many. The fulcrum being placed near one end, say an inch distant, at which end there is hung an ounce weight, then if the other arm be 16 inches long, a weight of one ounce hung at the distance of one inch, will just keep the lever or steel-yard in equilibrium. A weight of two ounces at the distance of two inches will do the same, so three ounces at the distance of three inches, and sixteen ounces, or one lb., at the distance of sixteen inches, &c.; so that, by moving the weight along the steel-yard till we get it to a balance, we may weigh any thing from an ounce

times for once that the larger one is turned round; so that the velocities of the wheels will be inversely as their numbers of teeth. In like manner, it is clear, that if the larger wheel drives the smaller not by teeth but by a band, their revolutions will be inversely as their circumferences.

Exam.—The number of teeth in one wheel are 160, and in another driven by it are 20, and the larger wheel makes 12 revolutions in a minute; how many does the smaller one make?

20 : 160 :: 12 : 96 = the number of turns

which the smaller wheel makes in a minute.

24. The larger wheel is usually called the wheel, driver, or leader, and the smaller one is called the pinion, driven wheel, or follower.

25. Let us now see what would be the action of two wheels and a pinion. If the first wheel contains 80 teeth, the pinion 12 teeth, and second wheel 36 teeth. Place the first wheel and the pinion on the same axis, so that they move together, one revolution of the one being in the same time as a revolution of the other, and the pinion drives the second wheel. If the first wheel makes 16 revolutions in a minute, the pinion will do the same, and the pinion drives the second wheel; therefore, 36 : 12 :: 16 : $5\frac{8}{36}$ = the velocity of the second wheel. Place these so, that the teeth of the first wheel act in the teeth of the pinion, and these again act in the teeth of the second wheel. If the first wheel makes as before 16 turns in a minute, then the pinion will make 12 : 80 :: 16 : $106\frac{8}{12}$ = in a minute; consequently, the revolutions of the second wheel will be 36 : 12 :: $106\frac{8}{12}$: 35·55 = turns of the second wheel in a minute.

26. When there are a number of wheels A, B, C, D, E, acting on the respective pinions *a*, *b*, *c*, *d*, *e*, as (fig. 87,) then the effect of the whole may be found thus: if the letters which represent the wheels and pinions be understood to signify the number of teeth of each,

$$\frac{\text{power} \times A \times B \times C \times D \times E}{a \times b \times c \times d \times e} = \text{weight.}$$

If the velocity of the first wheel be used instead of the power applied, then this rule will give the resulting velocity instead of the weight.

Exam.—If the numbers of the teeth of the wheels are 9, 6, 9, 10, 12, and those of the pinions 6, 6, 6, 6, 6; then the power applied being 14 lbs., we have

$$\frac{14 \times 9 \times 6 \times 9 \times 10 \times 12}{6 \times 6 \times 6 \times 6} = 104 \cdot 8 \text{ lbs.} = \text{the weight.}$$

And, by the remark under the rule, if the first make 14 revolutions in the minute, the speed of the last will be 104·8 in the same time.

The same rule will apply to the case where the wheels act on each other by ropes or straps, if the circumferences of the wheels and pinions are used for the number of teeth.

27. It often happens, in the construction of machinery, that two shafts must be connected by means of teethed wheels, in such a way, that the one shaft's velocity shall bear a certain proportion to that of the other shaft; and we must determine the number of teeth in each of the connecting wheels and pinions.

Take the respective numbers of teeth in the pinions at pleasure, and multiply all these together, and their product again by the number of turns that the one shaft is to make for one turn of the other shaft. Take, now, this product, and find all the numbers which will divide it without a remainder, or divide its divisors without a remainder—*always excepting the number* 1. Arrange all these in one line, and separate them into parcels or bands, each containing as many numbers, or factors (as they are called), as you please; but observing, that there must be as many bands as there are wheels required; then the product of the numbers in each band will give the number of teeth in the respective wheels.

Thus, if one shaft is to turn 720 times for another shaft's once, and there be interposed 4 pinions, one of which is fixed to the end of the one shaft, each pinion having 6 teeth or leaves: then, $6 \times 6 \times 6 \times 6 \times 720$; all the divisors or factors of which are 3, 2, 3, 2, 3, 2, 3, 2, 2, 2, 3, 5, 2, 2, 3; these divided into 4 bands at pleasure, give the number of teeth in the wheels. Thus,

$$\text{Either}\begin{cases} 2\times3\times5 = 30, \\ 2\times2\times2\times3 = 24, \\ 2\times2\times3\times3 = 36, \\ 2\times2\times3\times3 = 36, \end{cases} \quad \text{Or}\begin{cases} 3\times3\times5 = 45, \\ 3\times2\times2\times2\times2 = 48, \\ 3\times3\times2 = 18, \\ 3\times2\times2\times2 = 24. \end{cases}$$

28. As the subject of wheel work is of the greatest importance to mechanics, we shall resume it in a more advanced part of this work, where it may be more properly introduced.

THE PULLEY.

29. If a rope or string pass round the groove or rim of a wheel, moveable round an axle, with a power at the one end of the string or rope, and a weight at the other,—such a machine is called a Pulley. The axis of the pulley may be either fixed or moveable, (as in figs. 88 and 89.) If the axis of the pulley be fixed, it only serves to change the direction of the power's action; but if it be moveable, the power acts with an advantage of two to one.

30. If in a system of pulleys (as fig. 90), where each pulley is embraced by a cord, attached at one end to a fixed point, and at the other to the centre of the moveable pulley next above it, and the weight is hung to the lowest pulley; then the effect of the whole will be = the number 2 multiplied by itself, as many times as there are moveable pulleys in the system: thus, if there be 4 moveable pulleys, then

$2 \times 2 \times 2 \times 2 = 16$; wherefore, if the weight be one lb. it will be sustained by a power of one oz. avoirdupois.

31. When there are any number of moveable pulleys on one block, and as many on a fixed block, the pulleys are called Sheefs, and the system is called a Muffle; and the weight is to the power inversely as one is to twice the number of moveable pulleys in the system, or

$$\frac{\text{the weight to be raised}}{\text{twice the number of mov. pulleys}} = \text{the power.}$$

Exam.—In a muffle where each block has 4 sheefs, one block being fixed and the other moveable, a weight of 112 lbs. is to be raised; hence,

$$\frac{112}{8} = 14 \text{ lbs., the power required.}$$

If a power of 236 lbs. is to be applied to a tackle connected with two blocks of pulleys, one fixed, consisting of 6, and another moveable, of 5 pulleys; what weight can be raised?—(Here the rule above must be reversed.)

Therefore $236 \times 10 = 2360$ lbs., the weight.

REMARK.—In all the above cases of the pulley, the strings, cords, or ropes, are supposed to act parallel to each other; when this is not the case, the relation of power and weight may be found bv aup!ying the principle of the *parallelogram of forces;* thus, draw *ab* (fig. 91,) in the direction of the power's action and of that length, taken from a scale of equal parts, which expresses the quantity of that power; next, draw *bd* a perpendicular to the horizon, and from *a* draw *ad* parallel to *bc*, the direction of the string, which is fastened at *c*: then the power is to the weight, as *ba* is to *bd*; and the strain on the hook at *c*, is as *ad* to *db*,—these lines being all measured on the same scale of equal parts.

It may be farther observed, that the pulley is a species of lever of the second kind; where the point at which the string is fastened may be called the fulcrum; the axis of the pulley

the place of the weight, and the place of the power the other end of the lever; or, the diameter of the pulley may be reckoned the length of the lever, the weight being in the middle.

THE INCLINED PLANE.

32. When a power acts on a body, on an inclined plane, so as to keep that body at rest; then the power, the weight, and the pressure on the plane, will be as the length, the height, and the base of the plane, when the power acts parallel to the plane: that is (see fig. 92),

$$\left.\begin{array}{l}\text{The weight}\\ \text{The power}\\ \text{The pressure on the plane}\end{array}\right\}\text{ will be as }\left\{\begin{array}{l}AB\\ BC\\ AC\end{array}\right.$$

33. The force with which a body endeavours to descend down an inclined plane, is as the height of the plane.

When the power does not act parallel to the plane, then from the angle C of the plane (fig. 93), draw a line perpendicular to the direction of the power's action; then, the weight, the power, and the pressure on the plane, will be as AC, CD, AD.

When the line of direction of the power is parallel to the plane, the power is least.

34. If two bodies, on two inclined planes, sustain each other, by means of a string over a pulley, their weights will be inversely as the lengths of the planes.

35. In the exercises on inclined planes, it is often necessary to find the length of the base, or height, or plane; any two of these being given—and this is done on the principle stated in Geometry, that the hypothenuse^2 of a right-angled triangle (the length of the plane) is equal to the base^2 + height^2.

Exam.—The height of an inclined plane is 20 feet, and it length 100; what is the pressure on the plane of a weight of 1000 lbs.? — Here we must first ascertain the base, $(100^2 - 20^2)^{\frac{1}{2}} = 97{\cdot}98 =$ the base of the plane, and from what has been said above, 100 : 10000 :: 97·98 : 9798 the pressure upon the plane; also 100 : 20 :: 10000 : 2000, the power necessary to keep the body from rolling down the plane.

If a waggon of 3 cwt. on an inclined railway of 10 feet to the 100, be sustained by another on an opposite railway of 10 feet to 90 of an incline; what is the weight of the second waggon?—Here 100 : 90 :: 3 : 2·7 cwt. = the weight of the second waggon.

36. The space which a body describes upon an inclined plane, when descending on the plane by the force of gravity, is to the space which it would fall freely in the same time, as the height is to the length of the plane; and the spaces being the same, the times will be inversely in this proportion.

Exam.—If a body roll down an inclined plane 320 feet long, and 26 feet in height; what space will it pass down the plane in one second, by the force of gravity alone?

320 : 26 :: 16 : 1·3 = the answer.

This subject, as connected with railways, will be resumed when we come to treat of *friction*.

THE WEDGE.

37. The wedge is a triangular prism, formed either of wood or metal, whose great use is to split or raise timber, stones, &c.

The circumstances in which it is applied are so various, that it is not easy to devise a general rule to determine the amount of its action. The wedge has a great advantage over all the other mechanic powers, in consequence of the way

in which the power is applied to it, namely, by percussion, or a stroke, so that by the blow of a hammer, almost any constant pressure may be overcome.

THE SCREW.

38. The screw is a kind of continued inclined plane, being an inclined plane rolled about a cylinder—the height of the plane being the distance between the centres of two threads, and its length the circumference; hence, the rule to find the power of a screw pressing either upwards or downwards, is as the distance between two threads of the screw is to the circumference where the power is applied: thus, if the distance of the centres of two threads of the screw be $\frac{1}{4}$ of an inch, and the radius of the handspike attached to the screw be 24 inches; the circumference of the screw will be $150\frac{4}{5}$ inches, nearly: therefore, $\frac{1}{4} : 150\frac{4}{5} :: 1 : 603\frac{1}{5}$; and if the power applied be 150 lbs., the force of the screw will therefore be $603\frac{1}{5} \times 150 = 90480$ lbs.

39. Remarks on the mechanic powers.—These mechanic powers may be variously modified and applied, but still they form the elements of all machinery. In our calculations of their effects, we have not made allowance for *friction*, or the resistance arising from one body rubbing against another—a subject which will be discussed hereafter. The justice of the remark made before, will now be seen to hold generally, that of the two—velocity and power—whatever we gain in the one, we lose in the other; or, as power and weight are opposed to each other, there will always be an equilibrium between them, when the power $\times$ its velocity $=$ the weight $\times$ its velocity, that is, when the momentum of the effect is equal to the momentum of the cause.

All the advantage that we can obtain from the mechanic

powers, or their combinations, is to raise great weights, or overcome great resistances, *and this must be done at a loss of time;* or, to generate rapid velocities, as in turning-laths, or cotton-spinning machinery, *and this is done with a loss of power.*

MECHANICAL CENTRES.

1. THESE are the centres of gravity, oscillation, percussion, and gyration.

THE CENTRE OF GRAVITY.

2. There is a certain point in every body, or system of bodies connected together; which point, if suspended, the body or system of bodies will remain at rest when acted upon by the force of gravity alone;—this point is called the Centre of Gravity. If a body or system of bodies be suspended by any other point than the centre of gravity, such body or system of bodies will move round that point, until the centre of gravity be in a vertical line with the point of suspension. If a body be sustained from falling by two forces, the lines of direction in which these two forces act, will meet in the centre of gravity of the body, or, in the vertical line which passes through it.

3. It is often useful in calculation to consider the whole weight of a body as placed in its centre of gravity, but it is to be remembered, that gravity and weight do not signify the same thing,—gravity is the force by means of which, bodies, if left to themselves, fall to the earth in directions perpendicular to the earth's surface; weight, on the other hand, is the resistance or force which must be exerted, to prevent a given body from obeying the law of gravity.

4. To find the centre of gravity of any plane figure, mechanically: Suspend the figure by any point near its edge, and mark the direction of a plumb-line hung from that point, then suspend it from some other point, and mark the direction of the plumb-line in like manner. The centre of gravity of the figure will be in that point where the marks of the plumb-line cross each other. For instance, if we wish to find the centre of gravity of the arch of a bridge, we draw the plan upon paper to a certain scale, cut out the figure, and proceed with it as above directed; and by means of the plumb-line from the points of suspension, its centre of gravity will be found; whence, by measuring the relative position of this centre in the plan by the scale, we may determine by comparison its position in the structure itself.

5. We can find the centre of gravity of many figures by calculation.

6. The centre of gravity of a line, parallelogram, prism, cylinder, circle, circumference of a circle, sphere, and regular polygon, is the geometrical centre of these figures, respectively.

7. For a triangle,—draw a line from any angle to the middle of the opposite side, then $\frac{2}{3}$ of this line from the angle will be the position of the centre of gravity.

8. For a trapezium,—draw the two diagonals, and find the centres of gravity of each of the four triangles thus formed, then join each opposite pair of these centres of gravity, and these two joining lines will cut each other in the centre of gravity of the figure.

9. For the cone and pyramid,—the centre of gravity is in the axis, at a distance of $\frac{3}{4}$ of the axis from the vertex.

10. For the arc of a circle,—

$$\frac{\text{radius of circle} \times \text{chord of arc}}{\text{length of arc}} =$$

distance of the centre of gravity from the centre of the circle.

11. For the sector of a circle,—

$$\frac{2 \times \text{chord of arc} \times \text{radius of circle}}{3 \times \text{length of arc}} =$$

distance of the centre of gravity from the centre of the circle.

12. For a parabolic space,—the distance of the centre of gravity from the vertex is $\frac{3}{5}$ of the axis.

13. For a paraboloid,—the centre of gravity is $\frac{2}{3}$ of the axis from the vertex.

14. For two bodies,—if at each end of a bar a weight be hung, the common centre of gravity will be in that point which divides the bar, in the same ratio that the weights of the bodies bear to each other, and this point will be nearest the heavier body.

Examples.—If the line drawn from the middle of the base of a triangle to the opposite angle be 15, then we have $\frac{15}{3} \times 2 = 10 =$ the distance of the centre of gravity from the vertical angle.

If the height of a cone be 24 inches, then we have $\frac{24}{4} \times 3 = 18 =$ the distance of the centre of gravity from the vertex.

If the length of the arc of a circle be 157·07, and the chord 153·07, and radius 200; then,

$$\frac{200 \times 153{\cdot}07}{157{\cdot}07} = 194{\cdot}8 =$$

distance of the centre of gravity from the centre of the circle.

If there be the sector of a circle of which the chord, radius, and length of arc, are the same as in the last example, we have

$$\frac{2 \times 153{\cdot}07 \times 200}{3 \times 157{\cdot}07} = 87{\cdot}4 =$$

distance of the centre of gravity from the centre of the circle.

In a parabolic space, if the axis be 25 inches long, then $\frac{25}{5} \times 3 = 15 =$ the distance of the centre of gravity from the centre.

In a paraboloid, if the axis be 30, then we have $\frac{30}{3} \times 2 = 20$ = the distance of the centre of gravity from the vertex.

A bar of wood, 24 feet long, has a weight suspended at each end, that at one end being 16 lbs. and the other 4; then, we have

$$20 : 24 :: 16 : 19{\cdot}2$$
$$\text{and } 20 : 24 :: 4 : 4{\cdot}8$$

the distances of the weights from the common centre of gravity. The greater weight being least distant. Hence we see, that $19{\cdot}2 + 4{\cdot}8 = 24$, the whole length of the bar; and also $4 \times 19{\cdot}2 = 16 \times 4{\cdot}8 = 76{\cdot}8$; so that the principle of virtual velocities, stated before, holds good here also; and here it may be observed, that it is of the greatest importance to trace any leading principle of this kind, through its various applications, as it serves to link together and harmonize the whole, and enables us to apply and remember it with greater facility.

It is often necessary to determine the centre of gravity experimentally, as in many cases it cannot be conveniently done by calculation. To maintain the firmness of any body resting on a base, it is necessary that the perpendicular drawn from the centre of gravity of the body, to the base on which it rests, be within that base; and the body will be the more difficult to overset, the nearer that perpendicular is to the centre of the base, and the more extensive the base is, compared to the height of the centre of gravity.

THE CENTRE OF OSCILLATION.

1. The centre of oscillation in a vibrating body, is that point in the axis of vibration, in which, if the whole matter contained in the body were collected, and acted upon by the same force, it would, if attached to the same axis of motion,

perform its vibrations in the same time. The centre of oscillation is always situated in the straight line which passes through the centre of gravity, and is perpendicular to the axis of motion. It will be seen by these remarks, that the subject of *pendulums* must be considered here.

2. In theory, a *simple pendulum* is a single weight, considered as a point, hanging at the lower extremity of an inflexible right line, having no weight, and suspended from a fixed point or centre, about which it vibrates, or oscillates; a *compound pendulum*, on the other hand, consists of several weights, so connected with the centre of suspension, or motion, as to retain always the same distance from it, and from each other.

3. If the pendulum be inverted, so that the centre of oscillation shall become the centre of suspension, then the former centre of suspension will become the centre of oscillation, and the pendulum will vibrate in the same time: this is called the *reciprocity* of the pendulum; and it is a fact of the greatest utility, in experimenting on the lengths of pendulums.

4. Of the simple pendulum we may observe, that its length, when vibrating seconds, must in the first place be determined by experiment, as it vibrates by the action of gravity—which force differs at different distances from the pole of the earth. By the latest experiments, the length of the seconds' pendulum in the latitude of London, has been found to be 39·1393 inches, or 3·2615 feet; the length at the equator is nearly 39·027, and at the pole 39·197 inches. The length for the latitude of London may be taken for all places in Britain, without any material error.

5. The times of vibration of two pendulums, are directly proportional to the square roots of the lengths of these pendulums.

6. Thus: what will be the time of one vibration of a pendulum of 12 inches long at London?

$(39{\cdot}1393)^{\frac{1}{2}} : (12)^{\frac{1}{2}} :: 1 : 0{\cdot}5537 =$ time of one vibration.

If the pendulum be 36 inches long,

$(39{\cdot}1393)^{\frac{1}{2}} : (36)^{\frac{1}{2}} :: 1 : 0{\cdot}9590 =$ time of one vibration.

7. The lengths of the pendulums are to each other inversely as the squares of the numbers of vibrations made in a given time.

What is the length of a pendulum vibrating half-seconds, or making 30 vibrations in a minute?

$(60)^2 : (30)^2 :: 39{\cdot}1393 : 9{\cdot}7848 =$ length in inches.

8. All the rules for simple pendulums may be expressed as follows:

The time of one vibration in seconds of any pendulum is

$$= \frac{1}{\text{number of vibrations in one second}}$$

$$\text{or, } \left(\frac{\text{the length of the pendulum}}{39{\cdot}1386}\right)^{\frac{1}{2}}$$

Exam.—If the number of vibrations of a pendulum be 0·6257, then

$$\frac{1}{0{\cdot}6257} = 1{\cdot}598 = \text{the time of one vibration.}$$

Or, if the length of the pendulum be 100 inches, then

$$\left(\frac{100}{39.1386}\right)^{\frac{1}{2}} = 1{\cdot}598.$$

The length of a pendulum in inches $= 39{\cdot}1386 \times$ time of one vibration^2; or,

$$\frac{39{\cdot}1386}{\text{number of vibrations}^2}$$

The time of one vibration being, as we before found it, 1·598; then, $39{\cdot}1386 \times 1{\cdot}598^2 = 100 =$ the length of the pendulum. Or, if the number of vibrations in a second be as above, 0·6257, then we have

$$\frac{39{\cdot}1386}{{\cdot}6257^2} = 100.$$

The number of vibrations in a second may be found thus:

$$\frac{39{\cdot}1386}{\text{length of pendulum}} = \text{number of vibrations};$$

or, the number of vibrations in a second is equal to

$$\frac{1}{\text{time of one vibration.}}$$

If the time of one vibration be as above, ·6257, then

$$\frac{1}{\cdot 6257} = 1{\cdot}598 = \text{the time of one vibration;}$$

or, if the length be 100, we have

$$\left(\frac{39{\cdot}1386}{100}\right)^{\frac{1}{2}} = 1{\cdot}598.$$

When a clock goes too fast or too slow, so that it shall lose or gain in twenty-four hours, it is desirable to regulate the length of the pendulum so that it shall go right. The pendulum bob is made capable of being moved up or down on the rod by means of the screw. If the clock goes too fast, the bob must be lowered, and if too slow, it must be raised; and we have this rule: number of threads in an inch of the screw $\times$ the time in minutes that the clock loses or gains in 24 hours; this product divided by 37 will give the number of threads that the bob must be screwed up or down, so that the clock shall go right.

Exam.—If the rod have a screw 70 threads in the inch, and the pendulum is too long, so that the clock is 12 minutes slow in 24 hours; then we have

$$\frac{2 \times 70 \times 12}{37} = 45\tfrac{15}{37} = \text{threads we must raise the bob,}$$

so that the clock shall go right.

9. It is often desirable, that a pendulum should vibrate seconds, and yet be much shorter than 39·1393 inches; which may be done by placing one bob on the rod above the centre of suspension, and another below it: then, having the distances of the weights from the centre of suspension, we may find the ratio which the weights should bear to each other by this rule: Call D the distance of the lower, and d the distance of the upper weight, from the centre of suspension; then,

$$\frac{39 \cdot 1393 \times D - D^2}{39 \cdot 1393 \times d + d^2} =$$

a number which, when multiplied by the lower weight, will give the higher,—D and d are taken in inches.

Exam.—In a pendulum having two bobs, the one 12 inches below the centre of suspension, and the other 9·6 inches above the same centre, the lower weight being 8 ounces; what is the upper weight?

$$\frac{39 \cdot 1393 \times 12 - 12^2}{9 \cdot 6^2 + 39 \cdot 1393 \times 9 \cdot 6} = 0 \cdot 696.$$

hence, $0 \cdot 696 \times 8 = 5 \cdot 568$ ounces = the weight of the upper bob.

THE CENTRE OF PERCUSSION.

10. If a common walking-stick be held in the hand, and struck against a stone, at different points of its length, it will be found that the hand receives a shock when it is struck at any part of the stick, but at one particular point, at which, if the stick be struck, the hand will receive no shock—this point is called the centre of *percussion*, and is usually defined thus: The centre of percussion is that point in a body revolving about an axis, at which, if it struck an immoveable obstacle, all the motion of the body would be destroyed, so that it would incline neither way after the stroke.

11. The distance of the centre of percussion from the axis of motion, is the same as the distance of the centre of oscillation from the centre of suspension; and the same rules serve for both centres.

12. The distance of either of these centres from the axis of motion, is found thus:

13. If the axis of motion be in the vertex of the figure, and the motion be flatwise; then,

14. In a right line, or very thin cylinder $= \frac{2}{3}$ of its length;
In an isosceles triangle $= \frac{3}{4}$ of its height;
In a circle $= \frac{5}{4}$ of its radius;
In a parabola $= \frac{5}{7}$ of its height.

15. But if the bodies move sidewise, we have
In a circle $= \frac{3}{4}$ of the diameter;
In a rectangle suspended by one angle $= \frac{2}{3}$ of the diagonal.

16. In a parabola suspended by its vertex,

$$= \frac{\frac{5}{7}\text{ axis}}{\frac{1}{3}\text{ perimeter}}$$

but if suspended by the middle of its base $= \frac{4}{7}$ axis $+ \frac{1}{2}$ parameter.

17. In the sector of a circle $= \dfrac{3 \times \text{arc} \times \text{radius}}{4 \text{ chord}}$.

18. In a cone $= \frac{4}{5}$ axis $+ \dfrac{(\text{radius of base})^2}{5 \times \text{axis}}$

19. In a sphere $= \dfrac{2 \times \text{radius}^2}{5\,(d + \text{radius})} + \text{radius} + d$, where d is the length of the thread by which it is suspended.

20. We have given these rules for the sake of reference, but we shall illustrate by examples the most useful.

Examples.—What must be the length of a rod without a weight, so that when hung at one end it shall vibrate seconds?

To vibrate seconds, the centre of oscillation must be 39·1393 inches from that of suspension; hence, as this must be $\frac{2}{3}$ of the line, $2 : 3 :: 39{\cdot}1393 : 58{\cdot}7089 =$ the length of the rod.

What is the centre of percussion of a rod 46 inches long? $\frac{2}{3} \times 46 = 30\frac{2}{3}$ inches from the axis of motion.

In an isosceles triangle, suspended by one angle, and oscillating flatwise, the height is 24 feet; hence, $\frac{3}{4} \times 24 = 18 =$ the answer.

In a sphere the diameter is 14, and the string by which the sphere is suspended is 20 inches; therefore,

$$\frac{7^2 \times 2}{5(20+7)} + 7 + 20 = \frac{98}{135} + 27 = 27\text{·}726;$$

so that the centre of oscillation or percussion is farther from the centre than the sphere, by 7·726 inches.

THE CENTRE OF GYRATION AND ROTATION.

21. It will be seen, that the last two centres refer to bodies in motion round a fixed axis, and belonging to the same class; there is yet another centre to be considered, of the utmost importance to the practical mechanic. We saw, in determining the centre of oscillation, that we were finding a point in which, if all the matter of the body were collected, the motion would be the same as that of the body—which motion was caused by the action of gravity; but when the body is put in motion by some other force than gravity, the point in question becomes the centre of *gyration*. The centre of gyration may therefore be defined, that point in a body or system of bodies revolving round an axis, in which point, if all the matter in the body or system of bodies were collected, the same number of revolutions in a given time would be generated by the application of a given force, as would be generated by the same force applied to the body or system of bodies itself.

22. The position of the centre of gyration is a mean proportional between the centres of oscillation and gravity.

23. The centre of gyration of the following bodies may be found by these rules:

24. For a straight line or cylinder, whose axis of motion is in one end, = length × 0·5775.

25. For a cylinder or plane of a circle, revolving about the axis, or the circumference about the diameter, = radius × 0·7071.

26. For the plane of a circle about its diameter = ½ radius.

27. For the surface of a sphere about its diameter = radius × 0·8165.

28. For a solid sphere or globe, about its diameter = radius × 0·6324.

29. For the circumference of a circle upon a perpendicular axis passing through the centre = radius.

Exam.—What is the distance of the centre of gyration from the centre of motion, of a rod 58·7089 inches long? Here 58·7089 × ·5775 = 33·9044.

In a wheel of uniform thickness, revolving about its axis, the diameter is 36 inches; hence 18 × ·7071 = 12 = distance of the centre of gyration from the axis.

In a globe revolving about its diameter, which is 2 feet, the distance of the centre of gyration is = 12 × ·6324 = 7·5888.

30. Effects are proportional to their causes; the motion generated in any body is proportional to the force which produces that motion; hence we see, that all constant forces may be compared to the force of gravity. And it is often useful to know the time in which a revolving body of a certain weight, acted upon by a known constant force, will acquire a given velocity. The principles we have laid down in discussing the inclined plane, will here be found serviceable.

As the weight of the body to be moved,
Is to the weight or force causing it to move,
So is the length of an inclined plane, such, that the given force would just support the body upon it,
To the height of the plane.

Now, if in a wheel 6 feet diameter, whose weight, 400 lbs., is turned by a force of 56 lbs., acting at the distance of 18 inches from its centre of motion, its centre of gyration being 5 feet from the same centre; what will be the time required

to give by this force a velocity of 20 feet per second at the centre of gyration. Here, by the lever,

$$\frac{18 \times 56}{60} = 16\tfrac{48}{60} \text{ lbs.} =$$

the force exerted at the centre of gyration. We now wish to know the length of time in which a body would acquire a velocity of 20 feet per second, on an inclined plane, whose length is to its height as 400 : $16\frac{48}{60}$; wherefore, by the laws of falling bodies, we have

$$\frac{16\frac{48}{60}}{32} = \frac{16{\cdot}8}{32} = {\cdot}525,$$

the time required to fall perpendicularly; therefore, by the inclined plane, we have, 20 : 400 :: ·525 : 10·5 = the time required.

31. All the circumstances comprehended under this kind of rotatory motion, may be expressed by the following rules:

Let W express the weight of a wheel,
F, the force acting upon the wheel,
D, the distance of the force from the axis of motion,
G, the distance of the centre of gyration from the axis of motion,
t, the time the force acts,
v, the velocity acquired by the revolving body in that time.

$$\frac{G \times W \times v}{D \times t \times 32} = F \qquad \frac{G \times W \times v}{F \times t \times 32} = D$$

$$\frac{F \times D \times t \times 32}{W \times v} = G \qquad \frac{F \times D \times t \times 32}{G \times v} = W$$

$$\frac{G \times W \times v}{F \times D \times 32} = t \qquad \frac{F \times D \times t \times 32}{G \times W} = v$$

It is to be observed, before applying these rules, that the number of turns of a revolving body in a minute are often given, and it is required to find the velocity in feet per second. A wheel of 8 feet diameter, for instance, makes 12 revolutions in a minute; how many feet does a nail in its

circumference pass over in a second? Here, $8 \times 3{\cdot}1416 = 25{\cdot}1328$ feet the nail passes through in one revolution, but $25{\cdot}1328 \times 12 = 301{\cdot}5936 =$ the feet it passes through in a minute; hence, 60) 301·5936 (5·0265, the velocity per second. The whole may be expressed shortly thus:

$$\frac{8 \times 3{\cdot}1416 \times 12}{60} = 5{\cdot}0265.$$

Exam.—What must be the weight of a fly-wheel that makes 12 revolutions in a minute, whose diameter is 8 feet, urged by a force of 84 lbs. at its rim, acting for 6 seconds, the distance of the centre of gyration being 3 feet 6 inches?

$$\frac{84 \times 4 \times 6 \times 32}{3{\cdot}5 \times 5{\cdot}0265} = 3838 \text{ lbs.} = 1 \text{ ton, } 14 \text{ cwt. } 1 \text{ qr. } 2 \text{ lbs.}$$

In a wheel which is 2 tons weight, and 14 feet diameter, the centre of gyration is 6 feet from the centre of rotation, the velocity with which this wheel moves is 10 feet per second; what force must be applied for 8 seconds, at the distance of 3 feet from the centre, to generate that velocity? Here we have

$$\frac{6 \times 2 \times 10}{3 \times 8 \times 32} = \frac{120}{768} = {\cdot}1666 \text{ of a ton} = 3 \text{ cwt. } 1{\cdot}32 \text{ qr.}$$

What is the distance of the centre of gyration from the centre of motion of a fly-wheel, the force which moves the wheel being 2 cwt., acting at the distance of 7 feet from the centre of motion, and for 10 seconds, the weight of the wheel being $2\frac{1}{2}$ tons, and its velocity 8 feet per second? Here $2\frac{1}{2}$ tons $= 50$ cwt.

$$\frac{2 \times 7 \times 10 \times 32}{50 \times 8} = 1{\cdot}6 \text{ feet, distance of centre of gyration.}$$

What is the velocity acquired by a fly-wheel acted upon by a force of 84 lbs., at the distance of 4 feet from the axis, the time in which the force has been acting is 7 seconds, the weight of the wheel $1\frac{1}{2}$ tons, and the distance of the centre

of gyration 5 feet from the centre of motion? Here $1\frac{1}{2}$ ton = 30 cwt. = 3360 lbs.; therefore,

$$\frac{84 \times 4 \times 7 \times 32}{5 \times 3360} = 4{\cdot}4 \text{ feet per second} =$$

the velocity acquired by the wheel.

CENTRAL FORCES.

1. Intimately connected with the foregoing subject is that of *central forces*, the nature of which may be illustrated by a very simple instance. When a boy causes a stone in a sling to revolve round his hand, the stone is kept from falling off by the strength of the string, which continually draws the stone, as it were, to the hand which is the centre of motion; but if the string is let go, or breaks, then the stone will fly off in a straight line, by means of its *centrifugal force*; the strength of the string, while it restrains this tendency, is called the *centripetal force*: when both forces are spoken of they are jointly called *central forces*.

2. When a body revolves round a fixed centre, the centripetal force may sometimes be the cohesion of the particles of which the body is composed, or sometimes it may be the power of some attracting body—such as gravity in the case of the planets.

3. In talking of the angular velocity of a revolving body, we mean not the space which is passed over in a given time, but the number of degrees, minutes, &c., that the body describes in a certain time, whether the circle be large or small. Thus, a body moving in a circle of 10 feet diameter, may have an angular velocity of 15° in a second, so may also another body moving in a circle of 3 feet diameter; they will complete their respective circles in the same time, but the actual spaces they pass through are very different;

that is, their angular velocities are the same, but their actual velocities are not.

4. The central forces are as the radii of the circles directly, and the squares of the times inversely, also the squares of the times are as the cubes of the distances. When a body revolves in a circle by means of central forces, its actual velocity is the same as it would acquire by falling through half the radius by the constant action of the centripetal force. From these the following rules for central forces are derived.

5. $$\frac{\text{veloc. of rev. body}^2 \times \text{weight of body}}{\text{radius of revolution} \times 32} = \text{centrif. force.}$$

6. $$\frac{\text{veloc. of revol. body}^2 \times \text{weight of body}}{\text{centrifugal force} \times 32} =$$

radius of the circle of revolution.

7. $$\frac{\text{centrif. force} \times 32 \times \text{rad. circle}}{\text{veloc. of revolving body}^2} = \text{weight revol. body.}$$

8. $$\left(\frac{\text{rad. circle} \times 32 \times \text{centrifugal force}}{\text{weight}}\right)^{\frac{1}{2}} = \text{velocity.}$$

9. There will be no difficulty in applying what has been said to practice.

There are two fly-wheels of the same weight, one of which is 10 feet diameter, and makes 6 revolutions in a minute; what must the diameter of the other be to revolve 3 times in a minute? Here $6^2 : 3^2 :: 10 : 2{\cdot}5$ = the diameter of the second.

What is the centrifugal force of the rim of a fly-wheel, its diameter being 12 feet, and the weight of the rim 1 ton, making 65 turns in a minute.

$$\frac{12 \times 3{\cdot}1416 \times 65}{60} = 40{\cdot}84 =$$

the velocity in feet per second; hence,

$$\frac{40{\cdot}84 \times 1}{32 \times 6} = 8{\cdot}687 \text{ tons,}$$

the tendency to burst.

Let us employ the centre of gyration.—If the fly above mentioned is in two halves, which are joined together by bolts capable of supporting 4 tons in all their positions, the whole weight of the wheel is $1\frac{1}{2}$ tons, the circle of gyration is 5·5 feet from the axis of motion; what must be its velocity so that its two halves may fly asunder? The force tending to separate the two halves will be $\frac{1}{2}$ of the whole force; wherefore, by the rule,

$$\frac{32 \times 4 \times 5{\cdot}5 \times 2}{1{\cdot}5} = 30{\cdot}638 = \text{the velocity},$$

$11 \times 3{\cdot}1416 = 34{\cdot}5576 =$ circumference of circle of gyration, wherefore, 34·5576 : 30·638 :: 60 : 53·195 revolutions in a minute.

10. The steam-engine governor, or conical pendulum, acts on the principle of central forces. It is so constructed, that when the balls diverge, or fly outwards, the ring on the upright shaft is raised, and that in proportion to the increase of the velocity, squared—or the square roots of the distances of the ring from the top, corresponding to two velocities will be as these velocities.

Exam.—If a governor makes 6 revolutions in a second, when the ring is 16 inches from the top; what will be the distance of the ring when the speed is increased to 10 revolutions in the same time? The balls will diverge more, consequently the ring will rise and the distance from the top become less; therefore, we have

$$10 : 6 :: 16^{\frac{1}{2}} \text{ or } 4 : 2{\cdot}4$$

which, squared, gives 5·76 inches, the second distance of the ring from the top.

11. We shall elsewhere introduce other particulars on rotation and central forces.

STRENGTH OF MATERIALS.

Materials are exposed to four different kinds of strain:

1*st*. They may be torn asunder, as in the case of ropes and stretchers. The strength of a body to resist this kind of strain is called its Resistance to Tension, or Absolute strength.

2*nd*. They may be crushed or compressed in the direction of their length, as in the case of columns, truss beams, &c.

3*rd*. They may be broken across, as in the case of joists, rafters, &c. The strength of a body to resist this kind of strain is called its Lateral strength.

They may be twisted or wrenched, as in the case of axles, screws, &c.

Extensive and accurate experiments are necessary to determine the several measures of these strengths in the different materials; and when this is done, the subsequent calculations become comparatively easy. We shall therefore, in the first place, lay down the results of the experiments of practical men.

A.

TABLE OF THE FLEXIBILITY AND STRENGTH OF TIMBER.

Name of the wood.	U	E	S	C
Teak,	818	9657802	2462	15555
Poon,	596	6759200	2221	14787
English oak,	598	3494730	1181	9836
Do.	435	5806200	1672	10853
Canada oak,	588	8595864	1766	11428
Dantzic oak,	724	4765750	1457	7386
Adriatic oak,	610	3885700	1583	8808
Ash,	395	6580750	2026	17337
Beech,	615	5417266	1556	9912
Elm,	509	2799347	1013	5767
Pitch pine,	588	4900466	1632	10415

Name of the wood.	U	E	S	
Red pine,	605	7359700	1341	10
New English fir,	757	5967400	1102	9
Riga fir,	588	5314570	1108	10
Do.		3962800	1051	
Mar forest fir,	588	2581400	1144	9
Do.	403	3478328	1262	10
Larch,	411	2465433	653	
Do.	518	3591133	832	
Do.	518	4210830	1127	7
Do.	518	4210830	1149	7
Norway spar,	648	5832000	1474	12

Note.—The extensive use of the above table will be sh hereafter.

B.

Table showing the weight that will pull asunder a p one inch square.

Cast gold,	22000	Bismuth,	2
Cast silver,	41000	Good brass,	51
Anglesea copper,	34000	Ivory,	16
Swedish copper,	37000	Horn,	8
Cast iron,	50000	Whalebone,	7
Bar iron, ordinary,	68000		
Do. Swedish,	84000	COMPOSITIONS OF	
Bar steel, soft,	120000	Gold 5, copper 1,	50
Do. razor temper,	150000	Silver 5, copper 1,	48
Cast tin, Eng. block,	5200	Swed. copper 6, tin 1,	64
Do. grain,	6500	Block tin 3, lead 1,	10
Cast lead,	860	Tin 4, lead 1, zinc 1,	13
Antimony,	1000	Lead 8, zinc 1,	45
Zinc,	2600		

C.

The same from Rennie:

	Weight that would tear it asunder in lbs.	Length in feet that would break with its own weight.
Cast steel,	134256	39455
Swedish iron,	72064	19740
English iron,	55872	16938
Cast iron,	19096	6110
Cast copper,	19072	5092
Yellow brass,	17958	5180
Cast tin,	4736	1496
Cast lead	1824	306
Good hemp rope,	6400	18790
Do. one inch diam.	5026	18790

D.

The cohesive force of a square inch of iron; from different experimentalists.

Iron wire,	113077	English iron,	61600
Do.	93964	Do.	65772
Swedish Iron,	78850	Welsh iron,	64960
Do.	72064	Do.	55776
Do.	54960	French iron,	61001
Do.	53244	Russian iron,	59472
German iron,	69133	Cast iron,	68295
English iron,	66000	Do.	19488
Do.	55000	Welsh Do.	16255

E.

Table of the lateral strength of the following materials, one foot long, and one inch square.

	Weight that will break them.	Weight which they can bear with safety.
Cast iron,	3270 lbs.	1090 lbs.
Oak,	627 —	209 —
Memel fir,	390 —	130 —
American white pine,	206 —	69 —

F.

The force necessary to crush one cubic inch.

Aberdeen granite, blue, . . .	24556
Very hard freestone, . . .	21254
Black Limerick limestone, . .	19924
Compact limestone, ...	17354
Craiglieth stone, ...	15568
Dundee sandstone, ...	14919
Yorkshire paving stone, . . .	15856
Red brick, ...	1817
Pale red brick, ...	1265
Chalk, ...	1127

Cubes of one-fourth of an inch.

Iron cast vertically, ...	11140
—— horizontally, . . .	10110
Cast Copper, ...	7318
Cast tin, ...	966
Cast lead, ...	483

Having made these statements, we shall proceed to show, how by the assistance of theoretical results, they may be applied to the wants of the practical engineer.

a. The absolute strength of ropes or bars, pulled lengthwise, is in proportion to the squares of their diameters. All cylindrical or prismatic rods are equally strong in every part, if they are equally thick, but if not, they will break where the section is least.

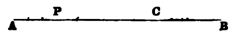
b. The lateral strength of any beam or bar of wood, stone, metal, &c., is in proportion to its breadth $\times$ its depth^2.—In square beams the lateral strengths are in proportion to the cubes of the sides, and in general of like-sided beams as the cubes of the similar sides of the section.

c. The lateral strength of any beam or bar, one end being fixed in the wall and the other projecting, is inversely as the distance of the weight from the section acted upon; and the strain upon any section is directly as the distance of the weight from that section.

d. If a projecting beam be fixed in a wall at one end, and a weight be hung at the other, then the strain at the end in the wall, is the same as the strain upon a beam of twice the length, supported at both ends and with twice the weight acting on its middle. The strength of a projecting beam is only half of what it would be, if supported at both ends.

e. If a beam be supported at both ends, and a weight act upon it, the strain is greatest when the weight is in the middle; and the strain, when the weight is not in the middle, will be to the strain when it is in the middle, as the product of the weight's distances from both ends, is to the square of half the length of the beam.—Take any two points in a beam supported at both ends; call one of these points *a*, and the other *b*; then a weight hung at *a* will produce a strain at *b*, the same as it would do at *a* if hung at *b*.

f. In a beam supported at the ends A and B, the strain at C, with the whole weight placed



there, is to the strain at C with the whole weight placed equally between C and P, as AC is to AP $\times$ $\frac{1}{2}$PC; and the strain at C by a weight placed equally along AP, is to the strain at C by the same weight placed on C, as $\frac{1}{2}$AP is to AC.

g. If beams bear weights in proportion to their lengths, either equally distributed over the beams or placed in similar points, the strains upon the beams will be as their lengths*.

h. If a beam rest upon two supports, and at the same time be firmly fixed in a wall at each end, it will bear twice as much weight as if it had lain loosely upon the supports; and the strain will be every where equal between the supports.

j. In any beam standing obliquely, or in a sloping direction, its strength or strain will be equal to that of a beam of the same breadth, thickness, and material, but only of the length of the horizontal distance between the points of support.

k. Similar plates of the same thickness, either supported at the ends or all round, will carry the same weight either uniformly distributed or laid on similar points, whatever be their extent.

l. The strength of a hollow cylinder, is to that of a solid cylinder of the same length and the same quantity of matter, as the greater diameter of the hollow cylinder is to the diameter of the solid cylinder; and the strength of hollow cylinders of the same length, weight, and material, are as their greater diameters.

m. The lateral strength of beams, posts, or pillars, are diminished the more they are compressed lengthwise.

n. The strength of a column to resist being crushed is directly as the square of the diameter, provided it is not so long as to have a chance of bending. This is true in metals or stone, but in timber the proportion is rather greater than the square.

o. The strength of homogeneous cylinders to resist being twisted round their axis, is as the cubes of their diameters; and this holds true of hollow cylinders, if their quantities of matter be the same.

PROBLEMS.

To find the strength of direct cohesion:

Area of transverse section *in inches* × measure of cohesion = strength in lbs. to resist being pulled asunder.

Exam.—In a square bar of beech, 3 inches in the side, we have $3 \times 3 \times 9912 = 59208$ lbs.

Note.—The measure of cohesion for timber is taken from col. C, table A, and for other materials from tables B or C.

In a beam of English oak, having four equal sides, each side being 4 inches, we have

$$4 \times 4 \times 9846 = 157536 \text{ lbs., the strength.}$$

In a rod of cast-steel 2 inches broad and 1½ inch thick, we have $2 \times 1\frac{1}{2} \times 134256 = 402768$ lbs., the strength.

What is the greatest weight which an iron wire $\frac{1}{10}$ of an inch thick will bear?

The area of the cross section of such wire will be ·007854, hence we have $·007854 \times 84000 = 659·736$ lbs.

To find the ultimate transverse strength of any beam:

When the beam is fixed at one end and loaded at the other, then the dimensions being in inches,

$$\frac{\text{breadth} \times \text{depth}^2 \times \text{transverse strength}}{\text{length of beam}} =$$

the ultimate transverse strength.

Note.—In column S, table A, will be found the transverse strength of timber, and in table E, that of iron, &c.; and let it be observed, that when the beam is loaded uniformly, the result of the last rule must be doubled.

What weight will break a beam of Riga fir, fixed at one end and loaded at the other, the breadth being 3, depth 4, and length 60 inches?

$$\frac{3 \times 4^2 \times 1108}{60} = 886\tfrac{2}{5} \text{ lbs.}$$

What weight uniformly distributed over a beam of English oak would break it, the breadth being 6, depth 9, and its length 12 feet?

$$\frac{6 \times 9^2 \times 1672}{144} \times 2 = 11286 \text{ lbs.}$$

If the number be taken from table F, we must use the length in feet.

When the beam is supported at both ends, and loaded in the centre,

$$\frac{\text{tabular value of S, tab. A} \times \text{depth}^2 \times \text{breadth} \times 4}{\text{length}} =$$

the weight in pounds.

Note.—When the beam is fixed at one end and loaded in the middle, the result obtained by the rule must be increased

by its half. When the beam is loaded uniformly throughout, the result must be doubled. When the beam is fixed at both ends and loaded uniformly, the result must be multiplied by 3.

Exam.—What weight will it require to break a beam of English oak, supported at both ends and loaded in the middle, the breadth being 6, and depth 8 inches, and length 12 feet?

$$\frac{1672 \times 8^2 \times 6 \times 4}{144} = 17834.$$

By using table E:

$$\frac{\text{depth}^2 \times \text{breadth} \times \text{tabular number}}{\text{length in feet}}$$

Exam.—What weight will a cast-iron bar bear, 10 feet long, 10 inches deep, and 2 inches thick, laid on its edge?

$$\frac{10^2 \times 2 \times 1090}{10} = 21800 \text{ lbs.}$$

The same on its broad side:

$$\frac{2^2 \times 10 \times 1090}{10} = 4360 \text{ lbs.}$$

To find the breadth to bear a given weight:

$$\frac{\text{length} \times \text{weight}}{\text{number in table F} \times \text{depth}^2} = \text{breadth.}$$

What must be the breadth of an oak beam, 20 feet long and 14 inches deep, to sustain a weight of 10000 lbs.

$$\frac{20 \times 10000}{14^2 \times 209} = 4{\cdot}85 \text{ inches} = \text{the breadth.}$$

To find the length:

$$\frac{\text{depth}^2 \times \text{breadth} \times \text{tabular number}}{\text{weight.}} = \text{length.}$$

In a beam 1 foot deep and 4 inches broad, the weight being 5000 lbs., then we have, if the beam be made of memel,

$$\frac{12^2 \times 4 \times 130}{5000} = 12{\cdot}77 \text{ feet, length required.}$$

To find the depth:

$$\left(\frac{\text{length} \times \text{weight}}{\text{tabular number} \times \text{breadth}}\right)^{\frac{1}{2}} = \text{depth}.$$

We wish to support a weight of 2000 lbs. by a beam of American fir; what is its depth, its length being 20 feet and breadth 4 inches?

$$\left(\frac{2000 \times 20}{69 \times 4}\right)^{\frac{1}{2}} = (145)^{\frac{1}{2}} = 12 \text{ inches, nearly.}$$

To find the deflection of a beam fixed at one end, and loaded at the other:

$$\frac{\text{length of beam in inches}^3 \times 32 \times \text{weight}}{\text{tab. numb. E } (\textit{in table } \text{A}) \times \text{breadth} \times \text{depth}^3} =$$

deflection in inches.

Note.—If the beam be loaded uniformly, use 12 instead of 32 in the rule.

If a weight of 300 be hung at the end of an ash bar fixed in a wall at one end, and 5 feet long, it being 4 inches square.

$$\frac{60^3 \times 32 \times 300}{6580750 \times 4 \times 4^3} = 1{\cdot}23 \text{ inches} = \text{the deflection.}$$

If the beam be supported at both ends and loaded in the middle:

$$\frac{\text{length (in inches)}^3 \times \text{weight}}{\text{tab. numb. (E, table A)} \times \text{breadth} \times \text{depth}^3} = \text{deflection.}$$

Note.—When the beam is firmly fixed at both ends, the deflection will be $\frac{2}{3}$ of that given by the rule.

Exam.—If a beam of pitch pine, 8 inches broad, 3 inches thick, and 30 feet long, is supported at both ends and loaded in the centre with a weight of 100 lbs.; what is its deflection?

$$\frac{360^3 \times 100}{4900466 \times 8 \times 3^3} = 4{\cdot}408 \text{ inches, deflection.}$$

If the beam had been firmly fixed at both ends, the deflection would have been

$$4{\cdot}408 \times \tfrac{2}{3} = 2{\cdot}935 \text{ inches.}$$

If the beam had been supported at both ends, and loaded uniformly throughout, the deflection would have been

$$4{\cdot}408 \times \tfrac{5}{8} = 3{\cdot}005.$$

To find the ultimate deflection of a beam of timber before it breaks:

$$\frac{\text{length } (\textit{in inches})^2}{\text{tab. numb. U (table A)} \times \text{depth}} = \text{ultimate deflection.}$$

What is the ultimate deflection of a beam of ash, 1 foot broad, 8 inches deep, and 40 feet long?

$$\frac{480^2}{396 \times 8} = 72{\cdot}91 \text{ inches, the ultimate deflection.}$$

To find the weight under which a column placed vertically will begin to bend, when it supports that weight:

$$\frac{\text{tab. numb. E (table A)} \times \text{least thickness}^3 \times \text{greatest} \times {\cdot}2056}{\text{length (in inches)}^2} =$$

weight in pounds. — It will be found by the application of this rule, that it will require 40289·22 lbs. to bend a beam of English oak 20 feet long, 6 inches thick, and 9 inches broad.

MODELS.

The subject of models deserves particular attention from the mechanic, from this circumstance, that although a model may be perfectly proportioned in all its parts as a model, yet the machine, if constructed in the same proportion, will not be sufficiently strong in every part; hence, particular attention should be paid to the kind of strain the different parts are exposed to; and from the statements which follow, the proper dimensions of the structure may be determined.

If the strain to draw asunder in the model be 1, and if the structure is 8 times larger than the model, then the stress

in the structure will be $8^3 = 512$. If the structure is 6 times as large as the model, then the stress on the structure will be $6^3 = 216$, and so on; therefore, the structure will be much less firm than the model; and this the more, as the structure is cube times greater than the model. If we wish to determine the greatest size we can make a machine having model, we have,

The greatest weight which the beam of the model can bear divided by the weight which it actually sustains = a quotient which, when multiplied by the size of the beam in the model, will give the greatest possible size of the same beam in the structure.

Exam.—If a beam in the model be 7 inches long, and bear a weight of 4 lbs., but is capable of bearing a weight 26 lbs.; what is the greatest length which we can make the corresponding beam in the structure? Here

$$\frac{26}{4} = 6{\cdot}5,$$

therefore, $6{\cdot}5 \times 7 = 45{\cdot}5$ inches.

The strength to resist crushing, increases from a model to a structure in proportion to their size, but the strain increases as before as the cubes; wherefore, in this case also, the model will be stronger than the machine, and the greatest size of the structure will be found by employing the square root of the quotient in the last rule, instead of the quotient itself; thus,

If the greatest weight which the column in a model can bear is 3 cwt., and if it actually bears 28 lbs., then, if the column be 18 inches high, we have

$$\left(\frac{336}{28}\right)^{\frac{1}{2}} = (12)^{\frac{1}{2}} = 3{\cdot}463;$$

wherefore, $3{\cdot}463 \times 18 = 62{\cdot}334$ inches, the length of the column in the structure.

SHAFTS.

The strength of shafts deserves particular attention; wherefore, instead of incorporating it with the general subject, *strength of materials*, we have allotted to it a separate chapter.

When the weight is in the middle of the shaft, the rule is

$$\left(\frac{\text{weight in lbs.} \times \text{length in feet}}{500}\right)^{\frac{1}{3}} = \text{diameter in inches.}$$

This is to be understood as the journal of the shaft, the body being usually square.

What is the diameter of a shaft 12 feet long, bearing a weight of 6 cwts., the weight acting at the middle?

$$\left(\frac{672 \times 12}{500}\right)^{\frac{1}{3}} = 2{\cdot}525 \text{ inches.}$$

If the weight had been equally diffused, we have, weight in lbs. × length; extract the cube root and divide by 10; the quotient is the diameter.

Thus, take the last example, then $672 \times 12 = 8064$; the cube root of which is 20·005, which divided by 10 gives 2·005, the diameter of the shaft.

If a cylindrical shaft have no other weight to sustain besides its own, the rule is, $(\cdot007 \times \text{length}^3)^{\frac{1}{2}} = \text{diameter}$; thus, a shaft having only the stress of its own weight is 10 feet long; hence

$(\cdot007 \times 10^3)^{\frac{1}{2}} = 2{\cdot}644$ the diameter of the shaft in inches.

For a hollow shaft supporting so many times its own weight, we have

$$\left(\frac{\cdot012 + \text{length}^3 \times \text{No. times its own weight}}{1 + \text{inner diameter}^4}\right)^{\frac{1}{3}} =$$

outer diameter in inches.

For wrought iron shafts find the diameter by the forego-

ing rules, which apply to cast iron, then multiply ·935, and for oak shafts the multiplier is 1·83, and for fir 1·716.

Exam.—What is the diameter of a cast iron shaft 12 feet long, and the stress it bears being twice its own weight? Here we have,

$$(\cdot 012 \times 12^3 \times 2)^{\frac{1}{2}} = 6\cdot 44 \text{ inches.}$$

For wrought iron, using the multiplier,

$$6\cdot 44 \times \cdot 935 = 6\cdot 0214,$$

and for oak, using the multiplier,

$$6\cdot 44 \times 1\cdot 83 = 11\cdot 3852,$$

and for fir, we have

$$6\cdot 44 \times 1\cdot 716 = 11\cdot 05104.$$

A rule often used in practice, though by no means a correct one, for determining the diameter of shafts is this. The cube root of the weight which the shaft bears taken in cwts. is nearly the diameter of the shaft in inches. It will be found safe in practice, to add one-third more to this result.

If a cast metal shaft has to bear a weight of $1\frac{1}{2}$ ton, that is, 30 cwts., then we have,

$$(30)^{\frac{1}{3}} = 3\cdot 107 \text{ inches by this rule;}$$

and, supposing it 12 feet long, we will apply the other rule, we have,

$$\left(\frac{3360 \times 12}{500}\right)^{\frac{1}{3}} = 4\cdot 319$$

We have now considered the strength of shafts, so far as regards their power to resist lateral pressure by weight acting on them; we have now to consider their power to resist torsion or twisting.

For cylindrical shafts, we have,

$$\left(\frac{240 \times \text{no. of horses' power}}{\text{no. of revolutions in a minute.}}\right)^{\frac{1}{3}} =$$

the diameter of the shaft in inches.

This rule is for cast iron; and it may be used for

wrought iron by multiplying the result by ·963, or for oak by 2·238, or for fir by 2·06.

If the shaft belong to a 7 horse power engine, and the strap turns 11½ in a minute,

$$\left(\frac{240 \times 7}{11 \cdot 5}\right)^{\frac{1}{3}} = 5 \cdot 267 \text{ inches diameter for cast iron.}$$

For fir, 5·267 × 2·06 = 10·85.

For oak, 5·267 × 2·38 = 12·535.

And for wrought iron, 5·267 × ·963 = 5·0719.

Note.—This rule comes from the best authority, and give perfectly safe results, though some employ 400, instead of 240, as a multiplier, which gives a greater diameter to the shaft. We may compare the two:

$$\left(\frac{400 \times 7}{11 \cdot 5}\right)^{\frac{1}{3}} = 5 \cdot 916$$

whereas the other was 5·267—something more than half an inch of difference.

It is to be remembered, that these rules relate to the shafts of first movers, or the shafts immediately connected with the moving power. But these shafts may communicate motion to other shafts, called second movers, and these again to others, called third movers, and so on. The diameters of the second movers may be found from those of the first, by multiplying by ·8, and these of the third movers, by multiplying by ·793, thus, if the diameter of the first mover be 5·267, then that of the second will be 5·267 × ·8 = 4·1936, and that of the third mover will be 5·267 × ·793 = 4·1767.

A material may resist one kind of strain much better than another, but expose them to a different kind of strain, and that which was weakest before may now be the strongest. This may be illustrated in the case of cast and wrought iron. The cast iron is stronger than the wrought iron when exposed to twisting or torsional strain, but the malleable iron is the stronger of the two when they are exposed to lateral pressure.

We shall subjoin a few results of experiments on the weight which was necessary to twist bars ¼ close to the bearings.

	lb.	*oz.*		*lb.*	*oz.*
Cast metal	9	17	English iron wrought,	10	2
Do. vertical cast,	10	10	Swedish iron wrought,	9	8
Cast steel,	17	9	Hard gun metal,	5	0
Sheer steel,	17	1	Brass bent,	4	11
Blister steel,	16	11	Copper cast,	4	5

It would appear that the strength of bodies to resist torsion, is nearly as the cubes of their diameters.

Remarks.—The rules and statements we have now given will often find their application in the practice of the engineer. On the proper proportioning of the magnitude of materials to the stress they have to bear, depends much of the beauty of any mechanical structure; and, what is of far greater moment, its absolute security. We will, in the appendix to this book, give some examples of the application of these principles to practice.

In the following table of the diameters of the shafts of first movers, the number of horses' power of the engine is given in the left hand column, and the number of revolutions the shaft makes in a minute is given in the top column. Then, to use the table, we have only to look for the power of the engine in the side column, and the number of turns the shaft makes in a minute in the line which runs across the top, and where these columns meet will be found the diameter of the shaft in inches. The table is constructed for cast iron, and first movers; the rules for finding the second and third have been given above, as also for finding equally strong shafts of other materials.

TABLE OF THE DIAMETERS OF SHAFT JOURNALS.

	10	20	30	40	50	60	70	80	90	100
5	5.9	4·7	4·1	3·7	3·5	3·3	3·1	3·0	2·9	2·7
6	6·3	5·0	4·4	4	3·7	3·5	3·4	3·2	3	2·9
7	6·6	5·2	4·6	4·2	3·9	3·6	3·5	3.4	3·3	3·1
8	6·9	5·5	4·8	4·4	4·1	3·9	3·7	3·5	3·4	3·3
9	7·2	5·7	5	4·5	4·2	4	3·7	3·6	3·5	3·4
10	7·4	5·9	5·2	4·7	4·4	4·1	3·9	3·7	3·6	3·5
15	8·5	7·0	6·0	5·5	5·1	4·6	4·5	4·3	4·2	4·0
20	9·3	7·4	6·6	5·9	5·6	5·2	5·0	4·6	4·5	4·4
30	10·7	8·4	7·4	6·9	6·5	5·9	5·7	5·5	5·2	5·0
40	11·7	9·5	8·3	7·4	6·9	6·6	6·2	5·9	5·7	5·6
50	12·6	10·0	9·0	8·0	7·4	7·2	6·8	6·5	6·2	5·9
60	13·6	10·8	9·3	8·6	7·7	7·4	7·2	6·8	6·7	6·4

This table answers for first movers only. It may, however, be made to give results, for second and third movers, by using the multipliers for that purpose, given above.

What is the diameter of the journal of the shaft of the first mover in a 30 horse power engine, the shaft making 40 revolutions in a minute? Here, by looking in the table, in the side column of horses' power, we find 30, and in the top column of revolutions, we find 40, and where these columns meet, we find 6·9 = the diameter of the first mover, in inches; wherefore, the second mover of this power and velocity will be = 6·9 × ·8 = 5·52 inches; and, in like manner, the third mover will be = 6·9 × ·64 = 4·416 inches = the diameter of the third mover to the same power and speed.

JOISTS AND ROOFS.

Joists should increase in strength in proportion to the squares of their lengths; for instance, a joist 16 feet long,

should be four times as strong as another joist 8 feet long, similarly situated; because, $8^2 : 16^2 :: 1 : 4$. From what has been previously stated, it will easily appear, that the stress on a beam or joist supported at both ends, increases towards the middle, where it is greatest; it therefore follows, that a beam should be strengthened in proportion to the increasing strain; and, as it would not be easy to add to the thickness of a beam towards the middle, which would destroy the levelness of the floor, a good substitute may be to fasten pieces to the sides of the joist, and thus increase its breadth, thus causing the beam to taper, in breadth, from the centre to the ends. In this way joists may be made much stronger than they usually are of the same length, and out of the same quantity of timber. It may also be observed, that joists are twice as strong when firmly fixed in the wall, as when loose, but it is to be remarked, that they have, when fixed, a far greater tendency to shake the wall. It is also to be remarked, that a joist is four times stronger when supported in the middle.

If the letter L represent the length of some known joist, whose strength has been tried, and D its depth, and T its thickness; and if another joist is required of equal strength with the former, when similarly situated; whose length is represented by l, its depth by d, and its thickness by t; we have the following rules:

1st. $\left(\dfrac{D^3 \times l^2}{L^2}\right)^{\frac{1}{3}} = d$ 2d, $\left(\dfrac{D^2 \times T \times l^2}{t \times L^2}\right)^{\frac{1}{2}} = d$

3d, $\dfrac{D^2 \times l^2 \times T}{d^2 \times L^2} = t$ 4th, $\left(\dfrac{d^2 \times t \times L^2}{D^2 \times T}\right)^{\frac{1}{2}} = l$

If a joist 30 feet long, 1 foot deep, and 3 inches thick, be sufficient in one case, what must the depth of a beam be, similarly placed, whose length is 15 feet, its depth and thickness bearing the same proportion to each other, as in the former beam? Here, by the first theorem, we have,

$$\left(\frac{1^2 \times 15^2}{30^2}\right)^{\frac{1}{3}} = (25)^{\frac{1}{3}} = {}^{\cdot}6298 \text{ feet} = 7{\cdot}55 \text{ inches}$$

the depth; and therefore 12 depth : 3 thickness :: 7·55 : 1·88 the breadth.

If the given beam be, as in the last example, 12 inches deep, 3 thick, and 30 feet long, and the required beam, of the same strength, is 8 inches deep, and 6 inches thick, then by the 4th we have,

$$\left(\frac{8^2 \times 6 \times 30^2}{12^2 \times 3}\right)^{\frac{1}{2}} = 28{\cdot}28 \text{ feet} = \text{length.}$$

If a joist, whose length is 30 feet, depth 12 inches, and thickness 8, is given, to find the depth of another of equal strength, only 6 inches thick, and 28·28 feet long? Here, by the 2d, we have,

$$\frac{12^2 \times 3 \times 28{\cdot}28^2}{30^2 \times 6} = 8 \text{ inches depth.}$$

To find the thickness from the same circumstances, we have by the 3d,

$$\frac{12^2 \times 3 \times 28{\cdot}28^2}{8^2 \times 30^2} = 6 \text{ inches the thickness.}$$

The same remarks hold true to a certain extent in roofing. A high roof is both heavier and more expensive than a low roof, as they will always be as the squares of the lengths of the couple-legs, so far as the scantling is concerned; but the slates and other materials increase in weight and expense as the length of the couple-legs simply. High roofs have, however, the advantage of being less severe upon the walls, than low ones, that is to say, so far as a tendency to push out the walls is concerned. To obtain the length of the rafter from that of the span, a common rule is to multiply the span by ·66 which gives the length of the rafter; thus, 14 feet of span gives 14 × ·66 = 9·24 feet the length of the rafter.

Note.—The numbers in the tables of the strength of materials are such as will break the bodies in a very short time; the prudent artist, therefore, will do well to trust no more

than about one-third of these weights; also great allowance must be made for knotty timber, and such as is sawn in any part across or oblique to the fibres.

WHEELS.

In page 120 we promised again to resume the subject of wheel-work, and we now proceed to consider, in the first place, the formation of the teeth of wheels.

A Cog-wheel is the general name for any wheel which has a number of teeth or cogs placed round its circumference.

A Pinion is a small wheel which has, in general, not more than 12 teeth, though, when two toothed wheels act upon one another, the smallest is generally called the pinion; so is also the trundle, lantern, or wallower.

When the teeth of a wheel are made of the same material and formed of the same piece as the body of the wheel, they are called *teeth;* when they are made of wood or some other material, and fixed to the circumference of the wheel, they are called *cogs;* in a pinion they are called *leaves;* in a trundle, *staves.*

The wheel which acts is called a *leader;* and the wheel which is acted upon by the former is called a *follower.*

When a wheel and pinion are to be so formed that the pinion shall revolve four times for the wheel's once, then they must be represented by two circles, whose diameters are to one another, as 4 to 1. When these two circles are so placed that they touch each other at the circumferences, then the line drawn joining their centres, is called the line of centres, and the radii of the two circles, the proportional radii.

These circles are called, by mill-wrights in general pitch-lines.

The distances from the centres of two circles to the extremities of their respective teeth, are called the real radii, and the distances between the centres of two contiguous teeth measured upon the pitch-line, is called the pitch of the wheel.

Two wheels acting upon one another in the same plane, are called *spur geer*. When they act at an angle, they are called *bevel geer*.

Teeth of wheels and leaves of pinions require great care and judgment in their formation, so that they neither clog the machinery with unnecessary friction, nor act so irregularly as to produce any inequalities in the motion, and a consequent wearing away of one part before another. Much has been written on this subject by mathematicians, who seem to agree that the epicycloid is the best of all curves for the teeth of wheels. The epicycloid is a curve differing from the cycloid described in page 94, in this, that the generating circle instead of moving along a straight edge, moves on the circumference of another circle.

The teeth of one wheel should press in a direction perpendicular to the radius of the wheel which it drives. As many teeth as possible should be in contact at the same time, in order to distribute the strain amongst them; by this means the chance of breaking the teeth will be diminished. During the action of one tooth upon another, the direction of the pressure should remain the same, so that the effect may be uniform. The surfaces of the teeth in working should not rub one against another, and should suffer no jolt either at the commencement or the termination of their mutual contact. The form of the epicycloid satisfies all these conditions; but it is intricate, and the involute of the circle is here substituted, as satisfying equally these conditions, and as being much more easily described.

Take the circumference of ABC of the wheel on which it is proposed to raise the teeth, (fig. 94) and let a be a point

from which one surface of one tooth is to spring, then fasten a string at A, such that when stretched and lying on the circumference shall reach to *a*; fix a pencil at *a*, and keeping the string equally tense, move the pencil outwards and it will describe the involute of the circle which will form the curve for one side of the tooth. Fasten the string at B so that its end, to which the pencil is fixed, be at the point from which the other face of the tooth is to spring—and proceed as above; then the curve of the other side of the tooth will be formed; and the figure of one of the teeth being determined, the rest may be traced from it.

The teeth of the pinion are formed in like manner.

The observation of practical men has furnished us with a method of forming teeth of wheels, which is found to answer fully as well in practice as any of the geometrical curves of the mathematician.

In fig. 95 we have the pattern of the segment of a wheel with the cogs fixed on in their rough state, and it is required to bring to their proper figure; they are consequently understood to be much larger than they are intended to be when dressed. The arc *b b* is the circumference of the wheel on which the bottoms of the teeth and cogs rest. Draw an arc *a*, *a*, on the face of the teeth for the pitch line of their point of action; draw also *d*, *d*, for their extremities or tops. When this is done, the pitch circle is correctly divided into as many equal parts as there are to be teeth. The compasses are then to be opened to an extent of one and a quarter of those divisions, and with this radius arcs are described on each side of every division on the pitch line *a*, *a*, from that line to the line *d*, *d*. One point of the compasses being set on *c*, the curve *f*, *g*, on one side of one tooth, and *o*, *n*, on the other sides of the other are described. Then the point of the compasses being set on the adjacent division *k*, the curve *l*, *m*, will be described; this completes the curved portion of the tooth *e*. The remaining portion

of the tooth within the circle *a, a*, is bounded by two straight lines drawn from *g* and *m* towards the centre. The same being done to the teeth all round, the mark is finished, and the cogs only require to be dressed down to the lines thus drawn.

It will be easy to determine the diameter of any wheel having the pitch and number of teeth in that wheel given. Thus, a wheel of 84 teeth having a pitch of 3 inches, we have $84 \times 3 = 162$ inches, the circumference, consequently,

$$\frac{162}{3{\cdot}1416} = 51{\cdot}5 \text{ inches diameter,}$$

or 4 feet $3\frac{1}{2}$ inches.

In the following table we have given the radii of wheels of various numbers of teeth, the pitch being one inch. To find the radius for any other pitch, we have only to multiply the radius in the table by the pitch in inches, the product is the answer. Thus for 30 teeth at a pitch of $3\frac{1}{2}$ inches, we have $4{\cdot}783 \times 3{\cdot}5 = 16{\cdot}74$ inches the radius.

0	0	1	2	3	4	5	6	7	8	9
0					0·636	0·795	0·954	1·114	1·273	1·432
10	1·668	1·774	1·932	2·089	2·247	2·405	2·563	2·721	2·879	3·038
20	3·196	3·355	3·513	3·672	3·830	3·989	4·148	4·307	4·465	4·624
30	4·783	4·942	5·101	5·260	5·419	5·578	5·737	5·896	6·055	6·214
40	6·373	6·532	6·643	6·850	7·009	7·168	7·327	7·486	7·695	7·804
50	7·963	8·122	8·231	8·440	8·599	8·753	8·962	9·076	9·235	9·399
60	9·553	9·712	9·872	10·031	10·190	10·349	10·508	10·662	10·826	10·935
70	11·144	11·303	11·463	11·622	11·731	11·940	12·099	12·758	12·417	12·676
80	12·735	12·895	13·054	13·213	13·370	13·531	13·690	13·849	14·008	14·168
90	14·327	14·436	14·645	14·804	14·963	15·122	15·281	15·441	15.600	15·759
100	15·918	16·072	16·236	16·395	16·554	16·713	16·873	17·032	17·191	17·350
110	17·504	17·668	17·987	17·827	18·146	18·305	18·464	18·623	18·782	18.941
120	19·101	19·260	19·419	19·578	19·737	19·896	20·055	20·214	20·374	20·533
130	20·692	20·851	21·010	21·169	21·328	21·488	21·647	21·806	21·460	22·124
140	22·283	22·442	22·602	22·761	22·920	23·074	23·238	23·397	23·556	23·716
150	23·875	24·034	24·193	24·352	24·511	24·620	24·830	24·989	25·148	25·307
160	25·466	25·625	25·784	25·944	26·103	26·262	26·421	26·580	26·739	26·894
170	27·058	27·217	27·376	27·535	27·694	27·853	27·931	28·172	28·331	28·490
180	28·699	28·808	28·967	29·126	29·286	29·445	29·604	29·763	29·922	30·086
190	30·241	30·400	30·559	30·718	30·877	31·036	31·196	31·355	31·514	31·673
200	31·832	31·992	32·150	32·310	32·469	32·628	32·787	32·846	33·105	33·264
210	33·424	33·583	33·742	33·901	34·060	34·219	34·278	34·537	34·697	34·856
220	35·015	35·174	35·333	35·492	35·652	35·811	35·970	36·129	36·288	36·447
230	36·607	36·766	36.925	37·084	37·243	37·402	37·561	37·720	37·880	38·039
240	38·198	38·357	38·516	38·725	38·835	38·994	39·153	39·312	39·471	39·631
250	39·790	39·949	40·108	40·262	40·426	40·585	40·744	40·904	41·063	41·222
260	41·381	41·541	41·699	41·858	42·019	42·177	42·336	42·495	42·654	42·813
270	42·973	43·132	43·291	43·450	43·609	43·768	43·927	44·087	44·231	44·405
280	44·564	44.723	44·882	45·042	45·201	45·360	45·519	45·678	45·837	45·996
290	46.156	46·315	46·474	46·633	46·792	46·751	47·111	47·270	47·429	47·588
300										

This will be found a very useful table in abridging calculation,—for instance, if we wish to find the radius of a wheel having 132 teeth, we look for 130 at the left-hand side column, and 2 at the top where these columns meet, we find the radius 21·010; and if the pitch of the wheels be $2\frac{1}{2}$, we multiply by it.

$21{\cdot}010 \times 2{\cdot}5 = 52{\cdot}525$ the radius of the required wheel.

The strength of wheels is a subject which has occupied the attention of the most eminent practical engineers, and the rules they have given us are entirely empirical, that is to say, the result of experiment.

The strength of the teeth will vary with the velocity of the wheel, the strength in horses' power at a velocity of 2·27 feet per second, will be

$$\frac{\text{breadth of the tooth} \times \text{its thickness}^2}{\text{length of tooth}} = \text{power.}$$

Required the strength in horses' power of a tooth 4 inches broad, 1·3 inches thick, and 1·6 inches long, at a velocity of 2·27 feet per second,—here we have

$$\frac{4 \times 1{\cdot}3^2}{1{\cdot}6} = 4{\cdot}15 = \text{the horses' power at a velocity of } 2{\cdot}27.$$

The power at any other velocity may be found by proportion, thus the same at 6 feet per second.

2·27 : 6 :: 4·15 : 10·9 = horses' power at a velocity of 6 feet per second.

The thickness of a tooth $\times$ 2·1 = the pitch.

The thickness of a tooth $\times$ 1·2 = length.

Ex.—The thickness of a tooth being 1·½ inches, then we have

$$1.5 \times 2{\cdot}1 = 3{\cdot}15 = \text{the pitch.}$$
$$1{\cdot}5 \times 1{\cdot}2 = 1{\cdot}8 = \text{the length.}$$

The breadth in practice is usually 2·5 times the pitch.

The arms of wheels generally taper from the axle to the rim, because they sustain the greatest stress towards the axle. It is obvious, that the more numerous the arms of a wheel are, they each suffer a proportionately less strain, as the resistance will be diffused over a greater number.

$$\frac{\text{The power acting at the rim} \times \text{length of arm}^2}{\text{number of arms} \times 2656 \times 0{\cdot}1} = \text{breadth}$$

and cube of depth.

Ex.—If the force acting at the extremity of the arm of a wheel be 16 cwt.; the radius of the wheel being 5 feet, and the number of arms 6, then we have 16 $\times$ 112 = 1792 lbs. = the force; wherefore,

$$\frac{1792 \times 5^3}{2656 \times 26 \times \cdot 1} = \frac{224000}{1593 \cdot 6} = 140 = \text{breadth}$$
and cube of depth.

Now, let us suppose that the breadth is 2 inches, we must divide this 140 by it, whence,

$$\frac{140}{2} = 70 = \text{the cube of the depth,}$$

and the cube root of 70 will be found $= 4 \cdot 121$, which is the depth of each arm.

When the depth at the axis is intended to be double of the depth at the rim, the number 1640 is to be used in the rule instead of 2656.

The tables which follow will be found in the highest degree useful to the practical mechanic.

EXPLANATION OF REFERENCES IN THE FOLLOWING TABLE.

[1] The only defect in this geering, which has been 16 years at work, is the want of breadth in the spur-wheel and pinion: they ought to have been 6 inches or more, as they will not last half so long as the bevel-wheels and pinions connected with them.

[2] Has been 16 years at work. The teeth were much worn.

[3] Has been 16 years at work. This geering was found rather too narrow for the strain, as it is wearing much faster than the rest of the wheels in the same mill.

[4] This wheel has wooden teeth, and has been working for three years.

[5] Ditto.

[6] This is a better pitch for the power than the following.

[7] This pitch has been found to be too fine.

TABLE OF PITCHES OF WHEELS IN ACTUAL USE IN MILL WORK.

Nature of the machinery.	Horses' power.	Pitch in inches.	Breadth of teeth in inches.	Wheel. Teeth.	Wheel. Revolves per minute.	Wheel. Diameter.	Pinion. Teeth.	Pinion. Revolves per minute.	Pinion. Diameter.	Breadth proportional to 10 horses' power, and present velocity.	Present velocity per second, in feet.	Breadth in inches proportional to 10 horses' power, at 3 f. p. second, that is, reducing all the examples to the same denom.
Horse-mill, ...	1	2¼	4	91	3	6..0½	22	12.9	1..5½	40·0	·949	12·65
Horse-mill, ...	1	2¼	4½	91	3	6..0½	20	13.13	1..4	45·0	·949	14·23
Water-wheel,[1]	5½	3	4	207		16..5¼	50		3..11⅝	7·27	·3	7·27
Water-wheel,[2]	10	4⅛	5½									5·5
Water-wheel, ...	15	3	6	204	4½	16..2	44	20	3..6	4·	3·8	5·06
Water-wheel,[3]	30	3	10¼	304	3⅐	24	3	318·47		3·41	3·95	4·489
Steam-engine,[4]	4	2¼	4¾	48	32	2..10⅕	25	61·11	1..6	11·87	4·8	18·99
Ditto, ditto,[5]	6	2¼	5¼	60	28	3..7	27	62·6	1..7⅜	8·75	5·25	15·31
Ditto, ditto,[6]	10	2¼	5¾	77	25	4..7⅛	40	48·5	2..4⅝	5·75	6·2	11·88
Ditto, ditto,[7]	10	1⅞	6	77	25	3..10	40	48·5	1..11⅛	6·	5·	10·
Ditto, ditto, ...	12	1⅜	3	66	44	2.8	48	60·5	1..9	2·5	5·99	4·95
Ditto, ditto, ...	12	2	4½	62		3..6			2..	3·75	5·8	11·
Steam engine, by B. & W. .	14	3	5	64	25	5..1	29	55	2..4	3·57	6·65	7·91
Steam engine, ...	20	2½	5	90	18	5..11	38	42·63	2..7	2·5	5·57	4·64
Steam engine by B. & W. .	24	3¼	6	96	19	8..0	42	43·32	3..6	2·5	7·95	6·625
Ditto, ditto, ...	32	3	6	116	19	8..10				1·87	8·78	5·47
Ditto, ditto, ...	46	3	8	152	17½		54	50		1·7	11·	6·2

EXPLANATION OF THE TABLE OF WHEELS ACTUALLY USED IN MILL-WORK.

The wheels are all reduced to what may be called one denomination,—

1st, By proportioning all their breadths to what they should be to have the same strength, if the resistance were equal to the work of a steam engine of ten horses' power.

2d, By supposing their pitch-lines all brought to the same velocity of three feet per second, and proportioning their breadths accordingly. This particular velocity of three feet per second has been chosen, because it is the velocity very common for overshot wheels.

Such cases as appear to have worn too rapidly, are marked, which may tend to discover the limit in point of breadth.

The following table of pitches of wheels was drawn up by Mr John Roberton, engineer, and is constructed in the following manner:—

The thickness of the teeth in each of the lines is varied one-tenth of an inch. The breadth of the teeth is always four times as much as their thickness. The strength of the teeth is ascertained by multiplying the square of their thickness into their breadth, taken in inches and tenths, &c. The pitch is found by multiplying the thickness of the teeth by 2. 1. The number that represents the strength of the teeth, will also represent the number of horses' power, at a velocity of about four feet per second. Thus, in the table where the pitch is 3·15 inches, the thickness of the teeth 1·5 inches, and the breadth 6 inches, the strength is valued at 13½ horses' power, with a velocity of four feet per second at the pitch line.

A Table of Pitches of Wheels, with the breadth and thickness of the teeth, and the corresponding number of horses' power, moving at the pitch line at the rate of three, four, six, and eight feet, per second.

Pitch in inches.	Thickness of teeth in inches.	Breadth of teeth in inches.	Strength of teeth, or no. of horses' power at 4 feet per second.	Horses' power at 3 feet per second.	Horses' power at 6 feet per second.	Horses' power at 8 feet per second.
3·99	1·9	7·6	27·43	20·57	41·14	54·85
3·78	1·8	7·2	23·32	17·49	34·98	46·64
3·57	1·7	6·8	19·65	14·73	29·46	39·28
3·36	1·6	6·4	16·38	12·28	24·56	32·74
3·15	1·5	6·	13·5	10·12	20·24	26·98
2·94	1·4	5·6	10·97	8·22	16·44	21·92
2·73	1·3	5·2	8.78	6·58	13·16	17·34
2·52	1·2	4·8	6·91	5·18	10·36	13·81
2·31	1·1	4·4	5·32	3·99	7·98	10·64
2·1	1·0	4·	4·0	3·0	6·0	8·0
1·89	·9	3·6	2·91	2.18	4.36	5·81
1.68	·8	3·2	2·04	1·53	3·06	3·08
1·47	·7	2·8	1·37	1.027	2·04	2.72
1·26	·6	2·4	·86	·64	1·38	1·84
1·05	·5	2·	·5	·375	·75	1·

HYDROSTATICS.

Hydrostatics comprehends all the circumstances of the pressure of non-elastic fluids, as water, mercury, &c., and of the weight and pressure of solids in them, when these fluids are at rest. Hydrodynamics, on the other hand, refers to the like circumstances of fluids in motion.

The particles of fluids are small and easily moved among themselves.

Motion or pressure in a fluid is not in one straight line in the direction of the moving force, but is propagated in every direction, upwards, downwards, sidewise, and oblique.

From this property it is that water will always endeavour to come to a level, for, if two cisterns be filled with water, the one 10 feet deep, and the other 6, there will be more pressure on the bottom of the 10 feet, than the 6 feet cistern; and, if the bottoms of both cisterns be on a level, and a pipe be made to communicate between them, then the water in the deep cistern will exert a greater pressure than that in the other, and will cause the other to rise till their pressures become equal, that is, when their surfaces come to a level; and this will hold true, however different the surfaces of the two cisterns may be in area. Hence, if water be communicated through pipes between any number of places, it will rise to the same level in all the places, whether the pipes be straight or bent, wide or narrow; and any fluid surface will rest only when that surface is level.

If a vessel contain water, the pressure on any point in the sides or bottom, is proportional to the perpendicular height of the fluid, above that point, in the side or bottom.

The pressure of a fluid upon a horizontal base, is equal to the weight of a column of the fluid, of the area of the base multiplied by the perpendicular height of the fluid, whatever be the shape of the containing vessel: so that by a long and very small pipe, the strongest casks or vessels may be burst asunder by the pressure of a very small quantity of water.

Exam.—Into a square box a tube is fixed, so that it shall stand perpendicularly; the area of the bottom of the box is 9 square feet, and the height of the top of the tube above the bottom of the box is 5 feet, and therefore the pressure on the bottom is $5 \times 9 = 45$ cubic feet of water. Now the weight of one cubic foot of water is found to be very nearly 1000 ounces, avoir., therefore, $45 \times 1000 = 45{,}000$ ounces, $= 1$ ton, 5 cwt. 2 qrs. 19 lbs.

The quantity of pressure upon any plane surface on which a fluid rests, is equal to the pressure upon the same plane placed horizontally at the depth of its centre of gravity.

If any plane surface, either vertical or inclined, be placed in a fluid, the centre of pressure of the fluid on the plane is at the centre of percussion, the surface of the fluid being supposed the centre of motion. Thus it will be found, that in a cistern whose sides are vertical, the centre of pressure on the sides is two-thirds from the top, which is also the centre of percussion.

To ascertain the whole pressure on a flood-gate, or other surface exposed to the pressure of water, a very near approach to the truth may be made by these rules—the breadth and depth being taken in feet.

$31{\cdot}25 \times \text{breadth} \times \text{depth}^2 = \text{pressure in lbs.}$

$\cdot 2727 \times \text{breadth} \times \text{depth}^2 = \text{pressure in cwts.}$

If the gate be wider at the top than bottom,

$$31{\cdot}25 \times \left(\frac{\text{breadth at top} - \text{breadth at bottom}}{3}\right) + \text{breadth}$$

at bottom $\times$ depth^2 = pressure in lbs.; and ·2727, used instead of 31·25, will give the pressure in cwts., nearly.

Exam.—What is the pressure upon a rectangular flood-gate, whose breadth is 25 feet, and depth below the surface of the water 12 feet; here,

$$31{\cdot}25 \times 25 \times 12^2 = 112500 \text{ lbs. pressure.}$$

If the breadth at top be 28 feet, that at bottom 22, and the height, as before, 12, then,

$$31{\cdot}25 \times \frac{28-22}{3} + 22 \times 12^2 = 107800 \text{ lbs pressure.}$$

The weight of a cubic foot of river water is about $\frac{9}{11}$ of a cwt. The pressure at the depth of 30 feet is about 13 lbs. to the square inch. And at the depth of 36 feet the pressure is about 1 ton to the square foot. The weight of an imperial gallon of water is about 10 lbs.

Exam.—What is the pressure at the depth of 120 feet on a square inch? 30 : 120 : : 13 : 52 = the pressure, and at the same depth 36 : 120 :: 1 : $3\frac{1}{3}$ tons on the square foot.

It is not difficult to see that the strength of the vessels or pipes which contain or convey water must be regulated according to the pressure.

The thickness of pipes to convey water must vary in proportion to the height of the head of water × diameter of pipe ÷ the cohesion of one square inch of the material of which the pipe is composed

By experiment it has been found that a *cast iron* pipe 15 inches diameter and $\frac{3}{4}$ of an inch thick of metal, will be sufficiently strong for a head 600 feet high. A pipe of oak 15 inches diameter and 2 inches thick, is sufficient for a head of 180 feet. When the material is the same, the thickness of the material varies with the height of head × diameter of pipe.

Exam.—What must be the thickness of a cast iron pipe 10 inches diameter for a head of 360 feet?

$$\frac{360 \times 10 \times \frac{3}{4}}{600 \times 15} = \frac{3}{10} \text{ of an inch thickness.}$$

If the same pipe is to be made of oak, then

$$\frac{360 \times 10 \times 2}{180 \times 15} = 2\tfrac{2}{3} \text{ thickness in inches.}$$

When conduct pipes are horizontal and made of lead, their thicknesses should be $2\frac{1}{2}$, 3, 4, 5, 6, 7, 8, lines, when the diameters are 1, $1\frac{1}{2}$, 2, 3, $4\frac{1}{2}$, 6, 7, inches—and when the pipes are made of iron, their thickness should be 1, 2, 3, 4, 5, 6, 7, 8, lines, when their diameters are 1, 2, 4, 6, 8, 10, 12.

The plumber should be aware that the tenacity of lead is increased four times, by adding 1 part of zinc to 8 of lead.

When the vessel which contains the water has, besides the pressure arising from the weight of the water, to resist an additional pressure exerted by some force on the water, as in Bramah's press, where the pressure exerted by means of a force pump on the water in a small tube, which communicates with a large cylinder, is, by the principles stated before in this chapter, multiplied on the portion of the cylinder as often as the area of the tube is contained in the area of the piston of the cylinder. If the area of the tube be one inch, the area of the piston 92 inches, then if the pressure on the water in the tube be 16 lbs. then the pressure on the piston will be $16 \times 92 = 1472$ lbs.

To ascertain the thickness of metal necessary for the cylinder of such presses, this rule will serve:

$$\frac{\text{pressure per square inch} \times \text{radius of cylinder}}{\text{cohesion of the metal per square inch—pressure}} =$$

thickness of metal necessary for the cylinder to sustain the pressure.

What is the thickness of metal in a press whose cylinder is 12 inches diameter, the pressure being 1·5 tons on the circular inch?

Note.—The cohesive force of a square inch of cast iron is 18000 lbs., and a circular inch is to a square inch in the ratio of 1 to 2·853, therefore, 1 : 2·853 :: 1·5 per circular

inch : 4·278 per square inch ; hence, by the rule,

$$\frac{4{\cdot}278 \times 6}{18000 - 4{\cdot}278} = 1{\cdot}87 \text{ inches,}$$

the thickness of metal.

FLOATING BODIES.

When any body is immersed in water, it will, if it be of the same density of the water, remain suspended in any place; but if it be more dense than the water it will sink, and if less dense it will float.

Bodies immersed and suspended in a fluid lose the weight of an equal bulk of the fluid, and the fluid acquires the weight that the body loses: also, bodies floating on a fluid lose weight in proportion to the quantity of fluid they displace.

When a body floats upon the water it will sink in the water, till the water which is displaced be equal in weight to the weight of the body.

When a body floats on a fluid, it will only be at rest when the centre of gravity of the body and the centre of gravity of the displaced fluid are in the same vertical line; and the lower the centre of gravity is, the more stable will the body be.

The buoyancy of casks, or the load which they will carry without sinking, may be estimated about 10 lbs. to the ale gallon, or 282 cubic inches of the content of the cask.

SPECIFIC GRAVITY.

Specific gravity is the relative weight of any body of a certain bulk, compared with the weight of some body taken as a standard of the same bulk. The standard of compari-

son is water; one cubic foot of which is found to weigh 1000 ounces, *avoir.* at a temperature of 60 of Fahrenheit, so that the weight expressed in ounces of a cubic foot of any body, will be its specific gravity, that of water being 1000.

To determine the specific gravity.

If the body be a solid heavier than water.—Weigh it first carefully in air, and note this weight; then immerse it in water, and in this state note its weight. Then divide the body's weight in air by the difference of the weights in air and water, the quotient is the specific gravity.

If the body be a solid lighter than water.—Tie a piece of metal to it, so that the compound may sink in water—then to the weight of the solid itself in air, add the weight of the metal in water, and from this sum subtract the weight of the compound in water, which difference makes a divisor to a dividend, which is the weight of the solid in air, then the quotient will be the specific gravity.

If the body be a fluid.—Take a solid, whose specific gravity is known, and that will sink in the fluid; then take the difference of the weights of the solid in and out of the fluid, and multiply this difference by the specific gravity of the solid; then, this product divided by the weight of the body in air, will give the specific gravity of the fluid.

On these principles there has been constructed tables of specific gravities, one of which we insert. The column, *specific gravity,* may be taken to represent the weight of a cubic foot, and the third column will give the weight of a cubic inch, both in ounces avoirdupois.

TABLE OF SPECIFIC GRAVITIES.

METALS.

	Specific Gravity.	Weight of a cubic inch in ounces Avoir.
Arsenic,	5763	3·335
Cast antimony	6702	3·878
Cast zinc,	7190	4·161
Cast iron,	7207	4·165
Cast tin,	7291	4·219
Bar iron,	7788	4·507
Cast nickel,	7807	4·513
Cast cobalt,	7811	4 520
Hard steel,	7816	4·523
Soft steel,	7833	4·533
Cast brass,	8395	4·858
Cast copper,	8788	5·085
Cast bismuth,	9822	5·684
Cast silver,	10474	6·061
Hammered silver,	10510	6·082
Cast lead,	11352	6·569
Mercury,	13568	7·872
Jeweller's gold,	15709	9·091
Gold coin,	17647	10·212
Cast gold, pure,	19258	11·145
Pure gold, hammered,	19361	11·212
Platinum, pure,	19500	11·285
Platinum, hammered,	20336	11·777
Platinum wire,	21041	12·176

Note.—All metals become specifically heavier by hammering.

STONES, EARTHS, &c.

	Specific Gravity.	Weight of a cub. foot in lbs. avoird.
Brick,	2000	125·00
Sulphur,	2033	127·08
Stone, paving,	2416	151·00
Stone, common,	2520	157·50

	Specific Gravity.	Weight of a cub. feet in lbs. avoird.
Granite, red,	2654	165·84
Glass, green,	2642	
Glass, white,	2892	
Glass, bottle,	2733	
Pebble,	2664	166·50
Slate,	2672	167·00
Marble,	2742	171·38
Chalk,	2784	174·00
Basalt,	2864	179·00
Hone, white razor,	2876	179·75
Limestone,	3179	198·68
RESINS, &c.		
Wax,	897	
Tallow,	945	
Bone of an ox,	1659	
Ivory,	1822	
LIQUIDS.		
Air at the earth's surface,	1¼	
Oil of turpentine,	870	
Olive oil,	915	
Distilled water,	1000	
Sea water,	1028	
Nitric acid,	1218	
Vitriol,	1841	
WOODS.		
Cork,	246	15·00
Poplar,	383	23·94
Larch,	544	34·00
Elm and new English fir,	556	34·75
Mahogany, Honduras,	560	35·00
Willow,	585	36·56
Cedar,	596	37·25
Pitch pine,	560	41·25
Pear tree,	661	41·31

	Specific Gravity.	Weight of a cub. foot in lbs. avoird.
Walnut,	671	41·94
Fir, forest,	694	43·37
Elder,	695	43·44
Beech,	696	43·50
Cherry tree,	715	44·68
Teak,	745	46·56
Mapple and Riga fir,	750	46·87
Ash and Dantzic oak,	760	47·50
Yew, Dutch,	788	49·25
Apple tree,	793	49·56
Alder,	800	50·00
Yew, Spanish,	807	50·44
Mahogany, Spanish,	852	53·25
Oak, American,	872	54·50
Boxwood, French,	912	57·00
Logwood,	913	57·06
Oak, English,	970	51·87
Do. sixty years cut,	1170	73·12
Ebony,	1331	83·18
Lignumvitæ,	1333	83·31

The above table will be found of the utmost use in determining the weight and magnitude of bodies.

To find the magnitude of a body from its weight:

$$\frac{\text{weight of body in ounces}}{\text{its specific grav. in table}} = \text{content in cubic feet.}$$

How many cubic feet are in one ton of mahogany? Here $20 \times 112 \times 16 = 35840$ ounces in a ton; therefore,

$$\frac{35840}{560} = 64 \text{ cubic feet.}$$

Had the timber been fir, then

$$\frac{35840}{556} = 64·45 \text{ cubic feet.}$$

Or English oak:

$$\frac{35840}{970} = 36·2 \text{ cubic feet.}$$

To find the weight of a body from its bulk:

cubic feet × specific gravity = weight in ounces.

What is the weight of a log of larch, 14 feet long, 2½ broad, and 1¼ thick? Here 2·5 × 1·25 × 14 = 43·75; wherefore,

43·75 × 544 = 23800 ounces = 13 cwt. 1 qr. 3 lbs. 8 oz.

What is the weight of a cast-iron ball, 2 inches diameter?

Here the content of the globe will be 2^3 × 5236 = 4.1888 cubic inches = ·039 feet, wherefore ·029 × 7207 =209 ounces = 13 lbs. nearly

A bullet of lead of the same magnitude would be be ·029 × 11352 × 329·4 ounces × 20·5 lbs.

If we wish to determine the quantity of two ingredients in a compound which they form,

Let H be the weight of the heavy body.

h, its specific gravity.

L the weight of the lighter body.

l, its specific gravity.

C, the weight of the compound.

c, its specific gravity.

then
$$\frac{(c - l) \times h}{(h - l) \times c} \times C = H$$

also
$$\frac{(h - c) \times l}{(h - l) \times c} \times C = L$$

Exam.—A mixture of gold and silver weighed 170 lbs. and its specific gravity was 15630; hence

h (by the table) = 19326. b = 10744

c = 15630 C = 170 lbs. wherefore, by the rule,

$$\frac{(19326-15630) \times 10744}{(19326-10744) \times 15630} \times 170 = \frac{39709824}{134136660} \times 170$$

= ·2888 × 170 = 50·096 lbs. of gold;

consequently there will be 170—50·096 = 119·904 lbs. of silver.

The weight of bodies—their magnitudes and also their

quantities in a compound, may thus be found by means of table of specific gravities; and for the more expeditious calculation in practice we add the following memoranda:

430·25 cubic inches of cast iron weigh one cwt., as also 397·60 of bar iron, 368·88 of cast brass, 352·41 of cast copper, and 372·8 of cast lead.

14·835 cubic feet of common paving stone weigh one ton, as also 14·222 of common stone, 13·505 of granite, 13·070 of marble, 64·46 of elm, 64 of Honduras mahogany, 51·65 of fir, 51·494 of beech, 42·066 of Spanish mahogany, and 36·205 of English oak.

For wrought iron square bars, allow 100 inches in length of an inch square to a quarter of a cwt.

A similar cast iron bar would require 9 feet in length for a quarter of a cwt. One foot in length of an inch square bar weighs $3\frac{1}{3}$ lbs. also the breadth and thickness being taken in, $\frac{1}{8}$th of an inch, and the length in feet.

$$\frac{\text{length} \times \text{breadth} \times \text{thickness} \times 7.}{144} = \text{the weight.}$$

in avoirdupois pounds.

Exam.—An iron bar 10 feet long, 3 inches broad, and $2\frac{1}{2}$ thick. Here 3 inches $=$ 24. and $2\frac{1}{2} =$ 20-8ths; therefore,

$$\frac{10 \times 24 \times 20 \times 7.}{144} = 233 \text{ lbs.}$$

For the weight of a cast iron pipe:

The length being taken in feet, the diameter and thickness of metal in inches, then we have

0·0876 $\times$ length $\times$ thickness $\times$ (inner diameter $+$ thickness) $=$ the weight in cwts.

For a leaden pipe the rule is,

0·1382 $\times$ length $\times$ thickness $\times$ (inner diameter $+$ thickness) $=$ the weight in cwts.

Note.—The weight of a cast iron pipe is to a leaden pipe of the same dimensions nearly as 7 is to 11.

Exam.—If the inner diameter or bore of a cast iron pipe

be 3 inches in its thickness; what is the weight of 14 feet of such pipe?

$\cdot 0876 \times 14 \times \frac{1}{4} \times (3 + \frac{1}{4}) = \cdot 99645$ cwt. $=$ 3 qrs. 27 lbs. 9 oz.

A leaden pipe is 12 feet long, the bore is 4 inches, and thickness of metal $\frac{1}{4}$ of an inch; therefore,

$\cdot 1382 \times 12 \times \frac{1}{4} \times (4 + \frac{1}{4}) = 1 \cdot 762$ cwt. $=$ 1 cwt. 3 qrs. 1 lb.

For the weight of the rim of a fly-wheel. Let D be the diameter of the fly, exclusive of the rim, taken in inches; then take the difference of this and the diameter of the fly, including the rim, and call this difference d, T being the thickness of the rim of the fly, from side to side, then we have

$\cdot 0073 \times T \times d \times (D + d) =$ the weight of the rim in cwts.

Exam.—If the interior diameter of the fly be 100 inches $=$ D, half the difference of the exterior and interior diameter $5 = d$, hence if the rim is 10 inches broad, as the exterior diameter will then be 110, and let the thickness of the rim be 4 inches $=$ T, then,

$\cdot 0073 \times 4 \times 5 \times (100 + 5)$ 15·33 cwts.

TABLE I.

Of the weight of 1 lineal foot of Swedish iron, of all breadths and thicknesses, from 1 tenth of an inch to 1 inch, in pounds and decimal parts.

·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0	10ths of Inches.
·034	·068	·101	·135	·169	·203	·237	·270	·304	·338	·1
	·135	·203	270	·338	·406	·473	·541	·608	·676	·2
		·304	·406	·507	·609	·710	·811	·913	1·014	·3
			·541	·676	·811	·947	1·082	1·217	1·352	·4
				·845	1·014	1·183	1·352	1·521	1·690	·5
					1·217	1·420	1·623	1·826	2·029	·6
						1·657	1·893	2·130	2·367	·7
							2·164	2·434	2·657	·8
								2·739	3·043	·9
									3·381	1·0

TABLE II.

Of the weight of 1 lineal foot of Swedish iron, of all breadths and thick nesses, from 1 inch to 6 inches, in pounds and decimal parts.

1	1¼	1½	1¾	2	2½	3	3½	4	5	6	In.
3.38	4·23	5·07	5·91	6·76	8·45	10·14	11·83	13·52	16·91	20·29	1
	5·29	6.34	7·40	8·45	10·56	12·68	14·79	16·91	21·13	25·36	1¼
		7·60	8·87	10·14	12·67	15·21	17·75	20·29	25·36	30·43	1½
			10·35	11·83	14·78	17·75	20·71	23·67	29·58	35·50	1¾
				13·52	16·91	20·29	23·67	27·05	33·81	40·51	2
					21·13	25·36	29·58	33·81	42·26	50·72	2½
						30·43	35·50	40·57	50·72	60·86	3
							41·42	47·34	59·16	71·00	3½
								54·10	67·62	81·14	4
									84·52	101·44	5
										121·72	6

TABLE III.

Of the Weight of 1 superficial Foot of Swedish Iron-Plate from 100th Part of an Inch thick to one Inch.

Thickness 100th parts of an inch.	Weight pounds and 100th parts.	Thickness 10th part of an inch.	Weight pound and 1000th parts.
·01	·406	·10	4·057
·02	·811	·2	8·114
·03	1·217	·3	12·172
·04	1·623	·4	16·232
·05	2·029	·5	20·286
·06	2·434	·6	24·344
·07	2·840	·7	28·401
·08	3·246	·8	32·458
·09	3·651	·9	36·516
·10	4·057	1·	40·573

TABLE IV.

Of Multipliers for the other Metals, whereby their Weights may be found from the above Tables.

Metals.	Multipliers.	Metals.	Multipliers.
Platinum laminated	2·846	Copper, cast . .	1·128
———, purified .	2·503	Brass wire . .	1·096
Pure gold, hammered	2·486	——, cast . .	1·080
———, cast .	2·472	Steel . . .	1·003
Lead . . .	1·457	Iron, Swedish .	1·
Pure silver, hammered	1·350	——, British . .	·980
——— cast .	1·344	——, cast . .	·925
Copper, wire . .	1·136	Pewter . . .	·960
———, hammered	1·132	Tin, cast . .	·937

TABLE V.

Table of the weight of one square foot of different metals of various thicknesses, in lbs. and Decimal parts.

Thickness in inches.	Mal. Iron, Swed.	Mal. Iron, English.	Cast Iron.	Copper.	Brass.	Lead.
$\frac{1}{16}$	2·535	2 486	2·345	2·860	2·738	3·693
$\frac{2}{16}$	5·070	4·972	4·690	5·720	5·476	7·386
$\frac{3}{16}$	7·605	7·458	7·035	8·580	8·214	11·079
$\frac{4}{16}$	10·140	9·944	9·380	11·440	10·952	14·772
$\frac{5}{16}$	12·675	12·130	11·725	14·300	13·690	18·465
$\frac{6}{16}$	15·216	14·916	14·670	17·160	16·428	22·158
$\frac{7}{16}$	17·851	17·402	16·415	20·020	19·166	25·851
$\frac{8}{16}$	20·280	18·288	18·760	22·880	21·904	29·544
$\frac{9}{16}$	22·815	22·774	21·105	25·740	24·642	33·237
$\frac{10}{16}$	25·350	24·260	23·450	28·600	27·380	36·930
$\frac{11}{16}$	27·885	26.746	25·795	31·460	30·118	40·623
$\frac{12}{16}$	30·410	39·232	28·140	34·320	32·856	44·316
$\frac{13}{16}$	32·945	1·728	30·485	37·180	35·594	48·009
$\frac{14}{16}$	35·4[illegible]0	34·214	32·880	40·040	38·332	51·702
$\frac{15}{16}$	38·015	36·700	35·225	42·900	41·170	55·405
1	40·550	39·186	37·570	45·760	43·908	59·098

The foregoing tables and rules will be found of the utmost service, in the ready calculation of the weight of materials commonly used in engineering. And, besides being highly serviceable in the construction of a machine, will enable the mechanic to form an estimate with greater accuracy and expedition.

What is the weight of a bar of Swedish iron 16 feet long, 3 inches broad, and 1·1 inches thick?

By table 2nd, 3·38 is the weight of one foot long, and one inch square, wherefore,

$3{\cdot}38 \times 16 \times 3 = 162{\cdot}24$; and in table 1st, we have for the weight of 1 foot by ·1 of an inch $=$ ·34, hence, $\cdot34 \times 3 \times 12 = 12{\cdot}17$; wherefore the sum of the two $= 162{\cdot}24 + 12{\cdot}17 = 174{\cdot}41$ lbs. the weight.

If we wish the weight of an equal bar of cast iron, we must employ the multipliers in table 4th; hence,

$174{\cdot}41 \times \cdot925 = 161.33$.

If we wished it for lead, the multiplier from the same table being 1·457, we have,

$174{\cdot}41 \times 1{\cdot}457 = 254{\cdot}115$ lbs., &c. &c.

Then if lead were 1 penny per pound, the price of such a bar would be.

$$\frac{254}{12} = 21\tfrac{2}{12} = £1\ 1\ 2$$

HYDRODYNAMICS.

As Hydrostatics embraces the consideration of fluids at rest, so hydrodynamics or hydraulics comprehends the circumstances of fluids in motion. Of this science, little, comparatively speaking, is yet known; but as it is of the utmost importance to man, we will endeavour to lay before our readers a statement of the more important results of modern inquiry.

If a fluid move through a pipe, canal, or river, of various breadths, always filling it, the velocity of the fluid at different parts will be inversely as the transverse sections of these parts.

The quantity of water that flows through a pipe, or in a canal or river, at any part, is in proportion to the area multiplied by the velocity at that part.

The calculation of the motion of rivers is often of the highest utility to the engineer. This is sometimes done by the employment of very intricate formulas, but such methods, if easier could be found, would evidently be inconsistent with the nature of this work. The method which we shall give is simple, and will be found to answer all the purposes of the practical man.

In a river the greatest velocity is at the surface and in the middle of the stream; from which it diminishes toward the bottom and sides where it is least.

The velocity at the middle of the stream may be ascertained by observing how many inches a body floating with the current passes over in a second of time. Gooseberries will fit this purpose exceedingly well, if they are not at hand, a cork may be employed.

Take the number of inches that the floating body passes over in one second, and extract its square root; double this

square root, and add one to the product, the result will be the velocity of the stream at the bottom.

And these velocities being ascertained, the mean velocity, or that with which if the stream moved in every part, would produce the same discharge, may be found = the velocity at top — velocity at top $^{\frac{1}{2}}$ + ·5.

Exam.—If the velocity at the top and in the middle of the stream, be 36 inches per second, then, $36 - 2 \times 36^{\frac{1}{2}} + 1 = 36 - 12 + 1 = 25 =$ the least velocity, or the velocity at bottom. And the mean velocity will be $= 36 - 36^{\frac{1}{2}} + \frac{1}{2} = 36 - 6 + \frac{1}{2} = 30{\cdot}5$.

When the water in a river receives a permanent increase from the junction of some other river, the velocity of the water is increased. This increase of velocity causes an increase of the action of the waters on the sides and bottom, from which circumstance the width of the river will always be increased, and sometimes, though not so frequently, the depth also. By reason of this increased action of the water on the bottom, the velocity is diminished, until the tenacity of the soil or the hardness of the rock afford a sufficient resistance to the force of the water. The bed of the river then changes only by very slow degrees, but the bed of no river is stationary.

It is of the greatest use to know the amount of the action of any stream on its bed, and for this purpose a knowledge of the nature of the bed and of the velocity at bottom, are absolutely necessary.

Every kind of soil has a certain velocity which will insure the stability of the bed. A less velocity would allow the waters to deposit more of the matter which is carried with the current, and a greater velocity would tear up the channel. From extensive experiments it has been found, that, a velocity of 3 inches per second at the bottom, will just begin to work upon the fine clay used for pottery, and, however firm and compact it may be, it will tear it up. A velocity of 6

inches will lift fine sand—8 inches, will lift coarse sand (the size of linseed)—12 inches, will sweep along gravel—24, will roll along pebbles an inch diameter—and 3 feet at bottom will sweep along shivery stones the size of an egg.

When water issues through a hole in the bottom or side of a vessel, its velocity is the same as that acquired by a body falling through free space from a height equal to that of the surface of the water above the hole.

The most correct rule for ascertaining the velocity of water running through pipes and canals is this:

$$\left(\frac{57 \times \text{height of head} \times \text{diam. of pipe}}{\text{length of pipe} + 57 \times \text{diam. of pipe}}\right)^{\frac{1}{2}} \times 23\tfrac{1}{3} =$$

the velocity in inches with which the water will issue from the orifice. All the measures are understood to be taken in inches.

Exam.—If there be a reservoir of water whose depth is 6 feet, having a tube 1 foot long and $2\frac{1}{2}$ inches; bore open so as to let the water escape at a distance of 18 inches from the bottom, then we have, $6 \times 12 = 72 =$ whole depth of water on the reservoir, and $72 - 18 = 54$, the height of the head of the fluid above the orifice, wherefore by the rule,

$$\left(\frac{57 \times 54 \times 2{\cdot}5}{12 + 57 \times 2{\cdot}5}\right)^{\frac{1}{2}} \times 23\tfrac{1}{3} = \left(\frac{7659}{1722}\right)^{\frac{1}{2}} \times 23\tfrac{1}{3} =$$

$(4{\cdot}4)^{\frac{1}{2}} \times 23\frac{1}{3} = 2{\cdot}097 \times 23\frac{1}{3} = 48{\cdot}86$ inches per second, the velocity of the water. And, by multiplying this result by the area of the orifice, we get the quantity discharged in one second—hence, if the pipe be circular, we have,

$$\frac{2{\cdot}5}{2} = 1{\cdot}25 = \text{radius, and } \frac{2{\cdot}5 \times 3{\cdot}1416}{2} = \text{half}$$

circumference $= 2{\cdot}108 =$ area of orifice, hence, $48{\cdot}86 \times 2{\cdot}108 = 101{\cdot}6$ cubic inches.

The quantity of water that flows out of a vertical rectangular aperture, that reaches as high as the surface, is $\frac{2}{3}$ of the

quantity that would flow out of the same aperture, placed horizontally at the depth of the base.

When water issues out of a circular aperture in a thin plate placed on the bottom or side of a reservoir, the stream is contracted into a smaller diameter, to a certain distance from the orifice. The vein is smaller at the distance of half the diameter of the orifice where the area of the section of the vein is $\frac{10}{16}$ of that of the orifice, and at the above point the stream has the velocity given by theory, so that to obtain the quantity of water discharged, we multiply the velocity by the area of the orifice, and $\frac{10}{16}$ of this will be the true result. When the water issues through a short tube, the vein of the stream will be less contracted than in the former case, in the proportion of 16 to 13. But when the water issues through an aperture which is the frustum of a cone, whose greater base is the aperture, the height of the conic frustum $=$ one half the diameter of the aperture and the area of the small end to that of the large end, as $10:16$; then, in this case, there will be no contraction of the vein; and from this it may be inferred, that, when a supply of water is required, the greatest possible from a given orifice, this form should be employed.

The resistance that a body sustains in moving through a fluid is in proportion to the square of the velocity.

The resistance that any plane surface encounters in moving through a fluid with any velocity, is equal to the weight of a column whose height is the space a body would have to fall through in free space to acquire that velocity, and whose base is the surface of the plane.

Exam.—If a plane 16 inches square, move through water at the rate of 13 feet per second; then,

$$\frac{13^2}{64} = 2{\cdot}6 =$$

the space a body would require to fall through free space to acquire a velocity of 13 per second, wherefore, as 2·6 feet =

31·2 inches, we have 16 × 31·2 = 499·2 cubic inches = the column of matter whose height and base are required; therefore, since 1728 cubic inches = 1 cubic foot of water weighs 1000 ounces, we have, 1728 : 499·2 :: 1000 : 110 ounces = 6 lbs. and a fraction, which is the amount of resistance met with by the plane at the above velocity.

As action and re-action are equal and contrary, it is the same thing whether the plane moves against the fluid, or the fluid against the plane.

WATER WHEELS.

Motion is generally obtained from water, either by exposing obstacles to the action of its current, or by arresting its progress during part of its descent, by moveable buckets.

Water-wheels have three denominations depending on their particular construction, undershot, breast, and overshot. If the water is to act on the wheel by its weight, it is delivered from the spout as high on the wheel as possible, that it may continue the longer to press the buckets down; but when it acts on the wheel by the velocity of the stream, it is made to act on the float-boards at as low a point as possible, that it may have acquired previously the greatest velocity. In the first case, the wheel is said to be overshot, in the second, undershot.

The overshot wheel is the most advantageous, as from the same quantity of water it gives a greater power, but it is not always that we can employ an overshot wheel from the smallness of the fall. When this is the case, we must deliver the water further down than the top of the wheel, and, in this case, it becomes a breast-wheel, and partakes in some degree of the properties of the overshot. When we cannot

employ a breast wheel, we must have recourse to the undershot, which is the least powerful of all.

Undershot wheels. The force of a stream of water against the floats of an undershot wheel is equal to a column of water, whose base is the section of the stream in that place, and height the perpendicular height of the water to the surface.

Where the quantity of water is given, its force against the floats of the wheel is directly proportional to the velocity of the stream, or the square root of the height of the surface. These remarks hold true only when the water is allowed to escape from the float boards, after it has struck them. For if the floats be too near each other, then the re-action of the water from one float will be sent back and obstruct the progress of the preceding float.

To find the velocity of the water acting upon the wheel, $(\text{height of the fall} \times 64{\cdot}38)^{\frac{1}{2}} =$ the velocity in feet per second.

Exam.—If the height of the fall be 14 feet, then we have $(14 \times 64{\cdot}38)^{\frac{1}{2}} = 30{\cdot}02$ feet per second.

To find the area of the section of the stream,

$$\frac{\text{The number of feet floating in 1 second}}{\text{velocity in feet per second}} =$$

the section of the stream in square feet.

Exam.—If there be 40 feet flowing in a second, and the velocity of the stream is 5 feet per second, then,

$$\frac{40}{5} = 8 =$$

the area of the section of the stream in square feet.

To calculate the power of the fall:

Area of section of stream where it acts upon the wheel $\times$ height of fall $\times$ $62\frac{1}{2} =$ the number of lbs. avoir. the wheel can sustain, acting perpendicularly at its circumference, so as to be in equilibrium. If this number of lbs. which keeps the wheel at rest be diminished, the wheel will move.

If the wheel move as fast as the stream, it is clear that the water would have no effect in moving it,—if the wheel were to move faster than the stream, the water would be a positive hindrance to its motion; and it can only be advantageous when the velocity of the stream is less than that of the wheel. And there is a certain relation between the velocity of the wheel and that of the stream, at which the effect will be the greatest possible or a maximum.

The effect of an undershot wheel is a *maximum* when the velocity of the wheel is $\frac{1}{3}$ of the velocity of the stream.

Exam.—If the area of the cross section of a stream be 6 feet, and its velocity 4 feet per second, and a fall of 16 feet can be procured, then we have, for the circumstances of an undershot wheel, $4 \times 6 = 24 =$ the number of cubic feet flowing per second:

$(16 \times 64{\cdot}38)^{\frac{1}{2}} = 32 =$ the velocity of the water at the end of the fall:

$\frac{24}{32} = \frac{3}{4} =$ the section of the stream at the end of the fall in square feet:

$\frac{3}{4} \times 16\ 62\frac{1}{2} = 750$ lbs. $=$ the weight which the wheel will sustain in equilibrium.

Now, the effective velocity of the stream is the difference between the velocities of the stream and wheel, and the wheel's velocity being $\frac{1}{3}$ of that of the stream, the difference on effective velocity will be $\frac{2}{3}$; now, the power of the stream is as the square of the effective velocity, and the square of $\frac{2}{3}$ is $\frac{4}{9}$. We must multiply the power of the fall as above calculated by this $\frac{4}{9}$, and also by $\frac{1}{3}$, in order that the wheel may move with the proper velocity; hence, $750 \times \frac{4}{9} \times \frac{1}{3} = 111\frac{1}{9}$ lbs. raised through $10\frac{2}{3}$ feet per second, the velocity of the wheel, which is $\frac{1}{3}$ of 32 the velocity of the stream.

An undershot water-wheel is capable only of raising $\frac{4}{27}$ of the weight of the water to the height of the fall.

From numerous experiments on water wheels, it has been

found, that in practice the water not being allowed to escape from the floats immediately after it has impinged upon them, the maximum effect is, when the velocity was between $\frac{1}{3}$ and $\frac{1}{2}$, that of the water being nearly $\frac{2}{2 \cdot 4}$. There is another deviation from theoretical result, in consequence of the water not being allowed to escape immediately from the floatboards, as the water is heaped up to about $2\frac{1}{2}$ times its natural height, and thus acts partly by its weight, and partly by its force—in consequence of which it happens, that a mill constructed undershot water wheel instead of raising $\frac{4}{27}$ of the weight of the water expended to the height of the fall will raise $\frac{1}{8}$

The effective head being the same, the effect of the wheel will depend on the quantity of water expended; and the quantity of water being the same, the effect of the wheel depends on the height of the head of the fall.

The section of the stream being the same, the effect will be nearly as the cube of the velocity.

Overshot water wheel.—If the water in the buckets of an overshot wheel be supposed to be equally diffused over half the circumference of the wheel, then the whole weight of the water in the buckets is to its power to turn the wheel as 11 : 7.

An overshot water wheel will raise nearly as much water to the height of the fall, as is expended in driving the wheel: if the height of the fall be reckoned from the bucket that receives the water to the bucket that discharges it.

According to the last experiments, the velocity of an overshot wheel, should be between 2 and 4 feet per second for all diameters of wheels.

Breast water wheel.—A breast wheel partakes of the properties of the two foregoing, as part of its action depends

on the velocity, and part on the weight of the water which moves it.

The power of a breast wheel is found, by taking that of an overshot, whose height is that from the receiving to the discharging buckets, added to that of an undershot whose head is the perpendicular distance between the receiving and discharging buckets.

Circumstances will regulate which of these three species of water wheel is to be employed. For a large supply of water with a small fall, the undershot wheel is the most appropriate. For a small supply of water with a large fall, the overshot ought to be employed. Where both the quantity of water and height of fall are moderate, the breast wheel must be used.

Before erecting a water wheel, all the circumstances must be taken into account, and our calculations made accordingly. We must measure the height of head velocity, and area of stream, &c., to do which a slight knowledge of levelling will be required.

LEVELLING.

A pole about 10 feet long must be procured, and also a staff about 5 feet long, on the top of which is fixed a spirit level with small sight holes at the ends, so that when the spirit level is perfectly horizontal, the eye may view any object before it through the sights in a perfectly horizontal line. If you have to measure the perpendicular distance between the bottom and top of a hill for instance; place the level staff on the side of the hill in such a way that when the level is truly set, the top of the hill may be seen through the sights. Keep the level in this position and look the contrary way, then cause some person to place the 10 feet staff before the

sight further down the hill, and looking through the sights to the staff, cause the person to move his finger up or down the staff until the finger be seen through the sights, and mark the position of the finger on the staff. Keep your 10 feet staff in the same place, and carry your level down the hill to a convenient distance, then fix it in the same way as before; and looking through the sights at the 10 feet staff, cause the person to move his finger up or down the staff in the same way until it be seen through the sights, and mark the place of the finger. Then the distance of the two finger marks added to the height of the level staff, will be the perpendicular distance between the place where the level staff now stands and the top of the hill. The process is perfectly simple, and it will not be difficult to repeat it oftener if the height of the hill requires it.

This process will give what is called the apparent level, which however is not the true level. Two stations are on the same true level when they are equally distant from the centre of the earth. The apparent level gives the objects in the same straight line, but the true level gives the line which joins them as part of a circle whose centre is the centre of the earth. In small distances there is no sensible difference between the true and apparent level of any two objects. When the distance is one mile, the true level will be about 8 inches different from the apparent level. This will serve well enough to remember, but more correctly speaking it is 7·962 inches for one mile, and for all other distances the difference of the two levels will be as the square of the distance. Thus at the distance of two miles it will be,

$1^2 : 2^2 :: 8 : 32$ inches, or 2 feet 8 inches.

These circumstances must be strictly observed in the formation of canals, rail-ways, &c. &c.

For the centre of gyration of a water wheel. Take the radius of the wheel and the weight of its arms, rim, shrouding, and float boards. Call the weight of the rim R, which must be multiplied by the square of the radius, and the product be doubled and then carried out. Next the weight of the arms called A must be multiplied by the square of the radius, and be doubled and carried out as before. Then the weight of the water in action called W must be multiplied by the square of the radius and carried out. If these products be added together into one sum they will form a dividend. For a divisor, double the weight of the rim and the arms, and add the weight of the water to them. Divide the dividend by the divisor, and the square root of the quotient will be the radius of gyration.

Exam.—In a wheel 24 feet diameter. The weight of the arms is 2 tons, the shrouding and rims 4 tons, and the water in action 2 tons; hence, by the above,

$$R = 4 \text{ tons} \times 12^2 \times 2 = 1152$$
$$A = 2 \text{ tons} \times 12^2 \times 2 = 576$$
$$W = 2 \text{ tons} \times 12^2 = 288$$

Their sum 2016 dividend, and

$$2 \times (4 + 2 + 2) = 16 \text{ the divisor.}$$

the answer, $\left(\frac{2016}{16}\right)^{\frac{1}{2}} = 126^{\frac{1}{2}} = 11{\cdot}225,$

THE MILLWRIGHT'S TABLE.

Tables for the more ready performance of calculations for water wheels are usually given in books of Mechanics: the construction and use of which we shall now proceed to show.

1. Find by measuring and levelling, the height of the fall of water which is reckoned from its upper surface to the middle of the depth of the stream, where it acts upon the float boards.

2. Find the velocity acquired by the water in falling through that height, which is done thus: multiply the height of the fall by 64·38, extract the square root of the product which would be the velocity of the stream if there were no friction, but to allow for friction take away $\frac{1}{20}$ of this result for the true velocity.

3. Find the velocity that ought to be given to the float boards, by taking $\frac{5}{7}$ of the velocity of the water which, product will be the number of feet the float boards have to press through in one second of time to produce the maximum effect.

$$\frac{\text{circumference of wheel}}{\text{velocity of the float boards}} =$$

the number of seconds that the wheel takes to make one turn.

4. Divide 60 by the last number. The quotient is the number of revolutions the wheel makes in one minute.

5. Divide 90 by the last quotient, the new quotient is the number of turns of the millstone for one of the wheel: 90 being the number of turns that a millstone of five feet diameter ought to make in a minute.

6. As the number of turns of the wheel in a minute
Is to the number of turns of the millstone in a minute,
So is the number of staves in the trundle
To the number of teeth in the spur-wheel, avoiding fractions.

7. The number of turns of the wheel in a minute, $\times$ the number of turns of the millstone for one of the wheels $=$ the number of turns of the millstone; or, the number of teeth in the spur-wheel $\times$ the number of turns of the water-wheel per minute, and this product divided by the number of staves in the trundle, the quotient is the number of turns of the millstone per minute.

In this way has the following table been constructed for a water-wheel of 15 feet diameter, the millstone being 5 feet diameter and making 90 turns in one minute.

A MILLWRIGHT'S TABLE,

In which the Velocity of the Wheel is Three-Sevenths of the Velocity of the Water, allowance being made for the Effects of Friction on the Velocity of the stream for a Wheel of Fifteen Feet Diameter.

Height of the Fall of the water.	Velocity of the Water per second.	Velocity of Wheel per Second, being 3-7th of that of the water.	Revolutions of the Wheel per Minute.	Number of Revolutions of the Millstone for 1 of the wheel.	Teeth in the Wheel, and Staves in the Trundle.		Revolutions of the Millstone per Minute by these Staves and Teeth.
Feet.	Feet. 100 Parts of a foot.	Feet. 100 Parts of a foot.	Revolutions. 100 Parts of a Revol.	Revol. 100 Parts of a Revol.	Teeth.	Staves.	Revol. 100 Parts of a Revol.
1	7·62	3·27	4·16	21·63	130	6	90·07
2	10·77	4·62	5·88	15·31	92	6	90·16
3	13·20	5·66	7·20	12·50	100	8	90·00
4	15·24	6·53	8·32	10·81	97	9	89·67
5	17·04	7·30	9·28	9·70	97	10	90·02
6	18·67	8·00	10·19	8·83	97	11	89·66
7	20·15	8·64	10·99	8·19	90	11	89·92
8	21·56	9·24	11·76	7·65	84	11	89·80
9	22·86	9·80	12·47	7·22	72	10	89·68
10	24·10	10·33	13·15	6·84	82	12	89·86
11	25·27	10·83	13·79	6·53	85	13	90·16
12	26·40	11·31	14·40	6·25	75	12	90·00
13	27·47	11·77	14·99	6·00	72	12	89·94
14	28·51	12·22	15·56	5·78	75	13	89·77
15	29·52	12·65	16·13	5·58	67	12	90·06
16	30·48	13·06	16·63	5·41	65	12	90·06
17	31·42	13·46	17·14	5·25	63	12	89·99
18	32·33	13·86	17·65	5·10	61	12	89·72
19	33·22	14·24	18·13	4·96	60	12	90·65
20	34·17	14·64	18·64	4·83	58	12	90·09

It is desirable that the millwright should possess short and easy rules which would answer the purposes of practice rather than the conditions of mere theory. The following will be found to answer the purpose, as they give the power with allowance for friction and waste of water.

For an undershot:

$$\frac{\text{Height of fall} \times \text{quantity of water flowing per minute}}{5000} =$$

the number of horses' power, the effect is equal to.

For an overshot:

Power of an undershot $\times\ 2\frac{1}{2}$ = horses' power.

For a breast wheel:

Find the power of an undershot from the top of the fall to where the water enters the bucket; then for an overshot for the rest of the fall—the sum of these two is the power of the breast wheel.

Note.—The quantity of water flowing per minute, and the height of the fall are both taken in feet.

Exam.—What power can be obtained from an undershot wheel. The fall being 25 feet, the section of the stream being 9 feet, and the velocity of the water 18 feet per minute.

$$\frac{9 \times 18 \times 25}{50000} = \frac{4050}{500} = \cdot 81 \text{ horse power,}$$

one horse power being unit.

And an overshot in the same situation would be $\cdot 81 \times 2\cdot 5 = 2\cdot 021$ horses' power.

And if in a breast wheel the water enters the bucket 12 feet from the top of the fall, then we have,

$$\frac{10 \times 8 \times 9}{5000} \times 2\tfrac{1}{2} = \frac{720}{5000} \times 2\tfrac{1}{2} = \frac{1800\cdot 0}{5000} = \cdot 36$$

for an overshot, and for the undershot we found it before; $\cdot 81$ hence, $\cdot 36 \times \cdot 81 = 1\cdot 17$ for the breast wheel.

BARKER'S MILL.

This ingenious machine has not been much employed, even in those situations for which it is best adapted; partly, we suspect, from the millwright's not having in his possession sufficiently simple rules for its construction; as the theory of this machine, simple as its construction and action may appear, is not by any means well developed. In the meantime, the following directions may be found useful to the mechanic.

1. Make each arm of the horizontal tube, from the centre of motion to the centre of the aperture of any convenient length, not less than $\frac{1}{6}$ of the perpendicular height of the water's surface above these centres.

2. Multiply the length of the arm in feet by ·61365, and that square root of this product will be the proper time for a revolution in seconds, and adapt the other parts of the machinery to this velocity; or,

If the time of a revolution be given, multiply the square of this time by 1·6296 for the proportional length of the arm in feet.

Multiply together the breadth, depth, and velocity per second of the race, and divide the last product, 14·27 × the square root of the height; the result is the area of either aperture;—or, multiply the continual product of the breadth, depth, and velocity of the race, by the square root of the height, and by the decimal ·07, the least product divided by the height will give the area of the aperture.

Multiply the area of either aperture by the height of the head of water, and this product by 55·79⁵ (or in round numbers 56) for the moving force, estimated at the centres of the apertures in lbs. avoirdupois.

Exam.—If the fall be 18 feet from the head to the centre

of the apertures, then the arm must not be less than 2 feet, as $\frac{1}{9}$ of $18 = 2$ and $(2 \times {\cdot}61365)^{\frac{1}{2}} = (1.22730)^{\frac{1}{2}} = 1{\cdot}107 =$ the time of a revolution in seconds: also, the breadth of the race being 17 inches, and depth 9, and the velocity of the water 6 feet per second, here we have,

$1{\cdot}24 \times {\cdot}75 \times 6 = 5{\cdot}58 =$ the area of section of the race $\times$ velocity of water; hence,

$5{\cdot}58 \times 18^{\frac{1}{2}} = {\cdot}07 = 1{\cdot}579 =$ the area of the aperture in inches; and,

$1{\cdot}579 \times 18 \times 56 = 1218$ lbs. the moving force.

The following dimensions have been employed in practice with success. The length of arm from the centre pivot to the centre of the discharging hole, 46 inches; inside diameter of the arm, 13 inches; diameter of the supplying pipe, 2 inches; height of the working head of water 21 feet above the level of the discharge. When the machine was not loaded and had one orifice open, it made 115 turns in a minute.

PNEUMATICS.

Pneumatics comprehends the knowledge of the properties of common air and elastic fluids in general.

Air is capable of being compressed to almost any degree, that is, may be forced into a space infinitely smaller than the space which it commonly occupies, and this is effected by additional pressure. When this additional pressure is taken away, the air will regain, by its elasticity, its former magnitude. Were it not for this circumstance, the subject of this chapter might have been introduced when we discussed the equilibrium and motion of water and fluids, which are nonelastic or incompressible, as their fundamental laws are the same. It has, indeed, been found by recent experimenters, that water, mercury, &c. are compressible, but to a very limited degree; so that although the distinction of elastic and nonelastic fluids is not absolutely correct, it is yet sufficiently so to retain Pneumatics, in elementary arrangement, as a distinct branch of science.

The air or atmosphere is a fluid body which surrounds the earth, and gravitates on all parts of its surface.

The mechanical properties of air are the same as other elastic fluids, and being the most common enquiries, in pneumatics are generally confined to this fluid.

The air has weight. A cubic foot of it weighs 1·2857 ounces at the surface of the earth, or, as some state it, 1·222.

The air being an elastic fluid, it is condensible and expansible, and its degrees of compression and expansion are proportional to the forces or weights which compress it.

All the air near the earth's surface is in a state of compres-

sion, in consequence of the weight of the atmosphere which is above it.

As the less weight that presses the air compresses it the less, or causes it to be less dense, and as the higher we rise in the atmosphere there will be the less weight, so the higher we go in the atmosphere the air will be the less dense.

The spring or elasticity of the air is equal to the weight of the atmosphere above it, and they will produce the same effects since they always sustain and balance each other.

If the density of the air be increased by compression, its spring or elasticity is also increased, and in the same proportion.

By the pressure and gravity of the atmosphere on the surface of fluids such as water, they are made to rise in pipes or vessels, where the spring or pressure within is taken off or diminished.

This fact, a knowledge of which is applied to a multitude of useful purposes, will not be difficult of explanation.

If a tube three feet long be filled with water, the tube being open at one end and close at the other. One unacquainted with the subject might naturally expect that if this tube were held perpendicularly with the open end downmost, the water would flow out of the tube by reason of its weight. But if we consider all the circumstances, we will see that this can only happen on certain conditions. The water has a tendency to fall to the earth in consequence of its weight, but then the air of the atmosphere, which we have stated before as also possessed of weight, and presses upon the surface of the water at the open end of the tube; and as the pressure of fluids of all kinds is exected in every direction, it follows, that the air will have a tendency to force the water up the tube. Now the pressure of the atmosphere at the surface of the earth is about 15 lbs. for every square inch, which is therefore the force by which the water will be pressed up the tube by the action of the air. A column of water 3 feet

high does not exert such a pressure on the base; wherefore, as the pressure upwards is greater than the pressure downwards, the water will remain suspended in the tube.

Let us now take a tube 36 feet long, similar to the former, filled with water and inverted in the same way as before, it will now be found that a part of the water will flow out of the tube, the reason of which will be easily seen. It was stated under Hydrostatics, that the pressure of a column of water 30 feet high was equal to 13 lbs. on the square inch. So that we see, that the pressure of the air will keep 30 feet of the water in the tube, but it will keep more, for the pressure of the air is 15, and that of 30 feet of water is only 13; and as the pressure of the water will be as its depth, we say, 13 : 15 : : 30 : 34, which, therefore, is the greatest height at which the water will be supported by the pressure of the atmosphere.

For the purpose of arriving at this conclusion of the effect of the pressure of the atmosphere, we might have employed a much shorter tube if we had used a heavier fluid than water, for instance, mercury. Now the cubic foot of mercury weighs 13600 ounces, and a cubic inch will be found,

$$\frac{13600}{1728} = 7 \cdot 866 \text{ ounces},$$

or nearly 8 ounces that is about half a pound avoirdupois; therefore 30 inches will weigh 15 lbs., hence, the atmosphere will balance by its pressure 30 inches of mercury.

Thus we have arrived at the principle of the barometer, or weather glass, as it is commonly called.

The pressure of the air at the surface of the earth is not always constant, but varies within certain limits. The mean pressure is about 14 lbs. to the square inch, and the corresponding height of the mercury in the barometer will therefore be 15 : 14 : : 30 : 28 inches.

It will appear evident, from what has been said before, that as the higher we ascend in the atmosphere there will be

less pressure, and therefore the mercury in the barometer will fall, and this fact has been used as a means of measuring heights by the barometer.

If the air were of the same uniform density up to the top of the atmosphere as it is at the earth's surface, we might very easily determine its height, for the specific gravity of air being to that of water as 1·222 to 1000, nearly, we have this proportion, 1·222 : 1000 : : 33·25, (the mean height of a water barometer in feet,) : 27600 feet, which is very nearly $5\frac{1}{4}$ miles; but by a process which proceeds on correct principles, the height of the atmosphere has been estimated at about 50 miles.

The law of the diminution of density at different heights in the atmosphere is this, that, if the heights increase in arithmetical progression, the densities will decrease in geometrical progression; for instance, if the density at the surface of the earth be called 1, and if at the height of 7 miles it be called 4 times rarer than at

14 16,
21 it will be 64 times rarer,
28 256,
35 1024,

and in this way it might be shown, that at the height of one-half diameter of the earth, one cubic inch of atmospheric air of the density which we breathe, would expand so much as to fill the bounds of the solar system.

When air becomes denser, its elastic force is increased, and that in proportion. Thus, when air is compressed into half its bulk, its elastic force will be double of what it was before.

It will, therefore, be easy to calculate the elastic force of air compressed any number of times;—thus, if, by any means, we condense the air in a vessel into $\frac{1}{3}$ of the space which it occupied when not confined, it will press on the in-

side of the vessel with a force of $15 \times 3 = 45$ lbs. on every square inch. It must be remembered, however, that the atmosphere presses with a force of 15 lbs. on each square inch of the outside of the vessel, which therefore counteracts so much of the force of the condensed air within—the real pressure, therefore, is $45 - 15 = 30$ lbs. It is clear, then, that whatever be the degree of condensation of the inclosed air, we must always deduct the pressure of the atmosphere to ascertain its true effect. The young mechanic will easily understand what is meant by the phrase—a pressure of 2, 3, 4, or any number of atmospheres, one atmosphere being understood as exerting a pressure of 15 lbs. on the square inch, two atmospheres 30, and three 45, &c.

When the air is by any means entirely taken out of any vessel, there is said to be a *vacuum* in that vessel.

It will be easy to see the mode of solution in the following questions.

What is the whole amount of pressure on the inside surface of a sphere, which contains air condensed to $\frac{1}{4}$ of its natural bulk, and is 6 inches in diameter within. Here, (by 17, page 100,) we have, $6^2 \times 3{\cdot}1416 = 113{\cdot}1976$ = the surface of the inside of the sphere—and $15 \times 4 - 15 = 45$ = the pressure on a square inch, therefore, $113{\cdot}1976 \times 45 = 5093{\cdot}892$ lbs. on the inner surface of the globe. Here the globe is supposed to be in a vacuum.

In a cylinder 6 feet long, and closed at the bottom, a piston is thrust down to the distance of one foot from the bottom, the cylinder being 24 inches in diameter, then, by the rules in mensuration, the area of the piston will be found to be 452·4 inches, the diameter of the piston being 24 inches, and the cylinder being 6 feet long, and the piston being pressed down to 1 foot from the bottom, the air will be compressed into $\frac{1}{6}$ of its former bulk, and its elastic force will be 6 times greater than it was before. At first it was 15 lbs. to the square inch, but now it will be $15 \times 6 = 90$ on the

square inch, and one atmosphere being deducted for the contrary pressure of the atmosphere above the piston, the pressure is $90 - 15 = 75$ lbs to the square inch, wherefore, $452 \cdot 4 \times 75 = 33830$ lbs., the force by which the piston will be pressed upwards.

We are now prepared to examine the action of several useful machines, whose operation depends on the pressure of the atmosphere.

THE SYPHON.

A syphon, or, as it is frequently written, siphon, is any bent tube.

If a syphon be filled with water and inverted, so that the bend shall be uppermost, then if the legs be of equal length, and it be held so that the two lower ends of the syphon are on a level, then we will find that if the perpendicular height of the bend of the tube above the level of the two ends be not more than 32, or from that to 34 feet, the water will remain suspended in the tube. It will not be difficult to see how this happens, for the atmosphere pressing on the water at the orifice of the tube at each extremity, presses the water up the tube with a force capable of raising it 34 feet; but in the case supposed, the orifices and the legs are equal, and do not exceed the limit of 32 or 34 feet, therefore, since the pressure on one orifice is the same as the pressure on the other, there will last an equilibrium—and the water in the one leg has no more power to move, than that in the other.

If we now suppose the syphon to be inclined a little, so that the two orifices shall not be on a level, or what is the same thing, if we suppose the length of the one leg to be greater than that of the other, we will find that the equili-

brium will be no longer maintained; and the water will flow out of the orifice which is lowest. For although the air presses equally on both orifices with a force of 15 lbs. to the square inch, yet the contrary pressures downwards by the weight of the water, are not equal, therefore motion will ensue where the power is greatest. If the shorter leg be immersed in a vessel of water, and the syphon be set running, the water will flow out of the lower end of the syphon, until the other end be no longer supplied. Instead of filling the syphon with water, as has been supposed above, a common practice is to apply the mouth to the lower orifice, and by sucking, exhaust the air in the tube, which diminishes the pressure at the other orifice, and consequently the action of the atmosphere will force the water in the vessel up the tube of the syphon and fill it, and it will continue to act in the same way as before.

We now proceed to the consideration of pumps.

PUMPS.

A pump is a machine used for raising water, sometimes by means of the pressure of the atmosphere, sometimes by the condensation of air, and sometimes by a combination of both.

It may be necessary here to explain what is meant by the term valve, that our remarks on the action of the pump may be rendered more intelligible.

A valve is usually defined to be a close lid affixed to a tube or opening in a vessel, by means of a hinge or some sort of moveable joint, and which can be opened only in one direction. There are various kinds of valves. The *clack* valve consists merely of a circular piece of leather covering the hole or bore of the pipe which it is intended to stop, and moving on a hinge, sometimes a part of itself, and sometimes

made of metal. The *butterfly* valve, which is superior to the clack valve, consists of two pieces of leather each formed into the shape of a half circle, they are attached by hinges on their diameters or straight parts to a bar that crosses the centre of the orifice to be closed. The *button* or *conical* valve consists of a plate of brass ground in such a way as exactly to fit the conical cavity in which it lies. A cylindrical tail or rod, which passes through a bar that lies across the bottom of the box, and is fixed at right angles to the under side of the valve. A little knob is placed at the underpart of the tail to prevent the valve from rising too far out of the socket. Sometimes valves are made in the form of pyramids consisting of four triangular flaps which form the sides of the pyramid, and move upon hinges which are placed round the edge of the orifice to be closed. The tops of these flaps must all meet accurately in the middle of the orifice, and are supported by four bars which meet in the centre.

We will now proceed to the consideration of the common *suction* pump. This pump consists of a hollow cylinder of wood or metal, which contains a piston stuffed so as to move up or down in the cylinder easily, and yet be air tight: to this piston there is attached a rod which will reach to the top of the cylinder when the piston is at the bottom. In the piston there is a valve which opens upwards, and at the bottom of the cylinder there is another valve also rising upwards, and which covers the orifice of a tube fixed to the bottom of the cylinder, and reaching to the well from whence the water is to be drawn. This tube is commonly called the *suction* tube, and the cylinder, the *body* of the pump.

From what has been said of the pressure of the atmosphere, it will not be difficult to understand how this machine operates. For when the piston is at the bottom of the cylinder, there can be no air, or at least very little between it and the valve, for as the piston was pushed down, the valve in it would allow the air to escape instead of being con-

densed, and when it is drawn up, the pressure of the air would shut the valve, and there would be a vacuum produced in the body of the cylinder when the piston arrived at the top. But the air in the suction tube would force up the valve at the bottom of the cylinder, and the air would thus be very much rarified, consequently the pressure of the air in the tube on the water at the bottom will be greatly less than that of the external atmosphere on the surface of the water in the well; therefore, the water will be pressed up the pump to a height not exceeding 32 or 33 feet. As the valves shut downwards, the water is prevented from returning, and the same operation being repeated, the water may be raised to any height, not exceeding the above limit, in any quantity.

The quantity of water discharged in a given time, is determined by considering that at each stroke of the piston a quantity is discharged equal to a cylinder whose base is the area of a cross section of the body of the pump, and height the play of the piston. Thus, if the diameter of the cylinder of the pump be 4 inches, and the play of the piston 3 feet, then, by mensuration, we have to find the content of a cylinder 4 inches diameter, and 3 feet high—now, 4 inches is the $\frac{1}{3}$ of a foot, or ·333, hence, $\cdot333^2 \times \cdot7854 = \cdot111999 \times \cdot7884 = \cdot08796$ = the area of the cross section of the cylinder in square feet; hence, $\cdot08796 \times 3 = \cdot2639$ = the content of the cylinder in cubic feet = the quantity in cubic feet of water discharged by one stroke of the piston. Now, a cubic foot of water weighs about 63$\frac{1}{2}$ lbs. avoirdupois, wherefore, $\cdot2639 \times 63\cdot5 = 15\cdot756$ lbs. avoir. and an imperial gallon is equal to 10 lbs. of water; whence, dividing the above number 15·087 by 10, we get the number of ale gallons = 1·5756. The piston, throughout its ascent, has to overcome a resistance equal to the weight of a column of water, having the same base as the area of the piston, and a height equal

to the height of the water in the body of the pump above the water in the well.

In our calculations of the effects of the pump, it will be necessary to determine the contents of pipes, for which purpose the following simple rules will serve.

Diameter of pipe in inches ^{2} = number of avoirdupois pounds contained in 3 feet length of the pipe.

If the last figure of this be pointed off as a decimal, the result will be the number of ale gallons, and if there be only one figure this is to be considered as so many tenths of an ale gallon: ale gallons $\times$ 282 = the number of cubic inches.

Thus, in a pipe 5 inches diameter, we have,

$5^2 = 25$ = number of avoirdupois pounds contained in 3 feet of the pipe $2{\cdot}5$ = the number of ale gallons and $2{\cdot}5 \times 282 = 705$ cubic inches.

These rules find the content for three feet in length of the pipe, the content for any other length may be found by proportion; thus, for a pipe 6 inches in diameter, and 12 feet long; we have, $6^2 = 36$ = pounds of water avoir. contained in the pipe to the length of 3 feet; therefore,

$3 : 12 :: 36 : 144$ = the number of pounds in 12 feet length, and,

$14{\cdot}4$ = ale gallons, and $14{\cdot}4 \times 282 = 4060{\cdot}8$ = the cubic inches in 12 feet length.

The resistance which is opposed to a pump rod in raising water, is equal to the weight of a column of water whose base is the area of the piston, and height the height of the surface of the water in the body of the pump above the surface of the water in the well, together with the friction and the piston and piston rod.

Suppose the body of the pump to be 6 inches in diameter, and the height to which the water is raised be 30 feet, and also the weight of the piston and rod is 10 lbs. and the friction is $\frac{1}{4}$ of the whole weight of the water.

Now, $6^2 = 36 =$ the lbs. avoirdupois of 3 feet of the column of water, but the column is 30 feet, therefore, 3 : 30 : : 36 : 360 lbs. the weight of the whole column. To this we must add the effect of friction, which we have supposed to be $\frac{1}{5}$ of the weight of the water; hence,

$\frac{360}{5} = 72$ lbs. and this must be added to the weight of the column of water, which gives $360 + 72 = 432$ lbs. the whole amount of resistance arising from the weight of the water and friction; to this must be added the weight of the piston and pump rod, therefore, $432 + 10 = 442 =$ the whole resistance opposed to the rising of the piston, any thing greater than this will raise it.

In the construction of pumps it is usual to employ a lever to work the piston, which gives an advantage in power; and if in the case estimated above, we employ a lever whose arms are in the proportion of 10 to 1, the pump might be wrought with a force of 44·2 lbs. or we may say 45 lbs.

For the convenience of workmen we insert the following table. It has been calculated on the supposition that the handle of the pump is a lever which gives an advantage on the piston rod of 5 to 1, and that a man can, with a pump 30 feet long, and a 4 inch bore, discharge 27·5 wine gallons (old measure) in a minute. And if it be required to find the diameter of a pump that a man could work with the same ease as the above pump at any required height above the surface of the well, this table will give the diameter of bore, and the quantity of water discharged in a minute.

Height of the pump above the surface of the mill in feet.	Diameter of the bore where the piston works in inches.	Water discharged per minute in wine measure, gallons and pints.	
10	6·93	81	6
15	5·66	54	4
20	4·90	40	7
25	4·38	32	6
30	4·	27	2
35	3·70	23	3
40	3·46	20	3
45	3·27	18	1
50	3·10	16	3
55	2·95	14	7
60	2·84	13	5
65	2·72	12	4
70	2·62	11	5
75	2·53	10	7
80	2·45	10	2
85	2·38	9	5
90	2·31	9	1
95	2·25	8	5
100	2·19	8	1

We stated before that water could not be raised to a greater height than 32 feet by means of the kind of pump we have described, and it may seem strange that this table extends to 100; but these are pumps acting on a different principle by means of which water may be raised to any height, and whose action will be considered before we leave this subject.

The *lifting* pump. This pump like the suction pump has two valves and a piston, both opening upwards; but the valve in the cylinder instead of being placed at the bottom of the cylinder is placed in the body of it, and at the height where the water is intended to be delivered. The bottom of the pump is thrust into the well a considerable way, and if the piston be supposed to be at the bottom, it is plain,

that as its valve opens upwards, there will be no obstruction to the water rising in the cylinder to the height which it is in the well; for by the principles of Hydrostatics, water will always endeavour to come to a level. Now when the piston is drawn up the valve in it will shut, and the water in the cylinder will be lifted up; the valve in the barrel will be opened and the water will pass through it, and cannot return as the valve opens upwards;—another stroke of the piston repeats the same process, and in this way the water is raised from the well: but the height to which it may be raised is not in this as in the suction pump limited to 32 or 33 feet. To ascertain the force necessary to work this pump, we are to consider that the piston lifts a column of water whose base is the area of the piston, and height the distance between the level of the water in the well and the spout, at which the water is delivered. Thus, if the diameter of the pump's bore be 4 inches, and the height of the spout above the level of the well $= 40$ feet, then we have $4^2 = 16$ lbs. in three feet of the barrel; wherefore,

$3 : 40 :: 16 : 213\frac{1}{3}$ lbs. the weight of the water, and the friction and weight of the piston and rod must be added to this to find the whole force necessary. If the friction be reckoned as it usually is $\frac{1}{5}$, then we have,

$$\frac{213}{5} = 42 \cdot$$

wherefore, $213 + 42 = 255$; as we have neglected fractions we may reckon it 256, and if the weight of the piston and rod be 20 lbs. the whole will be $256 + 20 = 276$ lbs. the whole force necessary to balance the piston, any thing greater than this will raise it.

The *forcing* pump remains now to be considered. The piston of this pump has no valve, but there is a valve at the bottom of the cylinder the same as in the first. In the side of the cylinder, and immediately above the valve, there is another valve opening outwards into a tube, which is bent upwards

to the height at which the water is to be delivered. When the piston is raised, the valve in the bottom of the pump opens, and a vacuum being produced, the water is pressed up into the pump on the principle of the sucking pump. But when the piston is pressed down, the valve at the bottom shuts, and the valve at the side which leads into the ejection tube opens, and the water is forced up the tube. When the piston is raised again this valve shuts, and the water cannot return. The same process is repeated, and the water is thrown out at every descent of the piston, the discharge therefore is not constant.

It is frequently required that the discharge from the pump should be continuous, and this is effected by fixing to the top of the eduction pipe an air vessel. This air vessel consists of a box, in the bottom of which there is a valve opening upwards into the box. This valve covers the top of the eduction pipe. A tube is fastened into the top of the box, which reaches nearly to the bottom of the box, it rises out of the box, and is furnished with a stop cock. If the stop cock be shut, and the water be sent by the action of the pump into the air vessel, it cannot return because of the shutting of the valve at the bottom of the box; and because of the space occupied by the water, the air in the box is condensed, and will consequently exert a pressure on the water in the air vessel. If the water fill three fourths of the box, then the air will be compressed so as to exert four times its original force; and the stop cock being opened the water will be forced up the tube, with a force which will send it one less than as many times 32 feet as the air is compressed, that is, in the case supposed $3 \times 32 = 96$ feet. On this principle it is that jets of fountains act.

The air vessel may therefore be considered as a magazine of power, and so long as there is as much water forced into the air vessel by pumping, as there is forced out by the pressure of the air, there will be a constant jet of water.

The force necessary to raise the piston in this pump, is found exactly in the same way as for the suction pump. And the force necessary to depress the piston, is found by taking the weight of a column of water, whose height is equal to the height of the spout where the water is delivered above the level of the piston, before it begins to descend. Thus, if the piston when raised is 26 feet above the level of the well, and the spout is 63 feet above the same level, therefore, the height of the column is $63 - 26 = 37$ feet; and supposing the diameter of the ejection pipe to be 5 inches, we have for 3 feet of the pipe $5^2 = 25$ lbs., wherefore for 37 feet we have,

$$3 : 37 :: 25 : 308\tfrac{1}{3} \text{ lbs.}$$

The weight of the piston and its rods oppose the raising of the piston, but assist in depressing it.

The power applied to the piston rod of a suction pump should be an intermitting power, otherwise there will be a waste of power; but in a forcing pump the power must be continued throughout equable. A single stroke steam engine will be best to raise water by the sucking, and a double stroke by a forcing pump. The piston rod of a forcing pump should be loaded with a weight sufficient to balance a column of water, whose base is the section of the piston, and whose height is the excess of the height of the spout from the level of the water in the cistern above 68 feet. This will cause a regular application of power when this pump is wrought with a steam engine.

WIND AND WIND-MILLS.

We have seen the effect of the pressure of air, arising from its weight and elasticity when at rest; it now remains for us to consider its effects when put in motion, as in the case of wind.

Were it not for the irregularity in direction and force of the wind, it would be the most convenient of all the first movers of machinery, but even as it is, its efficacy may be taken advantage of, and deserves our consideration.

The force with which wind strikes against a surface, is as the square of the velocity of the wind. This simple theorem is so nearly true that it may be employed without fear of error.

The force in avoirdupois pounds with which the wind strikes on any surface on which it acts perpendicularly may be found by using the rule,

$$\text{surface struck}^2 \times \text{velocity of wind}^2 \times \cdot 002288;$$

where the surface and velocity of wind are taken in feet, and the time 1 second. If the wind moves at the rate of 30 feet per second, and the surface exposed to its action be 14 feet square, then, $14 \times 30^2 \times \cdot 002288 = 28 \cdot 83$.

We here subjoin a tabular view of the effects of wind at different velocities.

From this statement it might appear at first sight, that in the case of wind-mills which act by the impulse of wind on revolving surfaces called sails—it might appear, we say, that the greater quantity of sail exposed to the action of the wind, the greater would be the effect of the machine. But this has been found not to hold: it would appear that the wind requires space to escape. The sails of the wind-mill may be supposed to intercept a cylinder of wind; and it would appear, that when the whole cylinder is intercepted, the effect of the machine is diminished; and it is concluded from experiments, that the sails should not intercept above seven eighths of the cylinder of wind.

TABLE

Showing the Pressure of the Wind for the following Velocities, from the Philosophical Transactions of the Royal Society of London.

Velocity of the Wind.		Force upon 1 square foot in Pounds Avoir.
Miles in 1 hour.	Feet in 1 second.	
1	1·47	·005
2	2·93	·020
3	0·40	·044
4	5·87	·079
5	7·33	·123
10	14·67	·492
15	22·00	1·107
20	29·34	1·968
25	36·67	3·075
30	44·01	4·429
35	51·34	6·027
40	58·68	7·873
45	66·01	9·963
50	73·35	12·300
60	88·02	17·715
80	117·36	31·490
100	146·70	49·200

The wind does not act perpendicularly on the sails of a windmill, but at a certain angle, and the sail varies in the degree of its inclination at different distances from the centre of motion, in resemblance to the wing of a bird; this is called the weathering of the sail. The angles of weathering have been found by Smeaton as follows. The radius being divided into 6 equal parts, and the first part from the centre being called 1, the last 6.

Distance from the centre.	Angle with the axis.	Angle with the plane of motion.
1	72	18
2	71	19
3	72	18
4	74	16
5	77½	12½
6	83	7

Smeaton's maxims for the construction of wind-mills.

1. The velocity of the wind-mill sails, whether unloaded or loaded, so as to produce a maximum, is nearly as the velocity of the wind, their shape and motion being the same.

2. The load at the maximum is nearly but somewhat less than, as the square of the velocity of the wind, the shape and position of the sails being the same.

3. The effects of the same sails at a maximum are nearly but somewhat less than, as the cubes of the velocity of the wind.

4. The load of the same sails at the maximum is nearly as the squares, and their effects as the cubes of their number of turns in a given time.

5. When the sails are loaded so as to produce a maximum at a given velocity, and the velocity of the wind increases, the load continuing the same, then, when the increase of the velocity of the wind is small, the effect will be nearly as the squares of the velocities; but when the velocity of the wind is double, the effects will be nearly as 10 to 27½; and when the velocities compared are more than double of that where the given load produces a maximum, the effect increases only as the increase of the velocity of the wind.

6. If sails are of a similar figure and position, the number of turns in a given time will be inversely as the radius of length of the sail.

7. The load at a maximum that sails of a similar figure and position will overcome at a given distance from the centre of motion, will be as the cube of the radius.

8. The effect of sails of similar figure and position are as the square of the radius.

Rules for modelling the sails of wind-mills.

Fig. 96 is the front view of one sail of a wind-mill. The length of the arm AA, called by workmen the whip, is measured from the centre of the great shaft B, to the outermost bar 19. The breadth of the face of the whip A next the centre, is $\frac{1}{30}$ of the length of the whip, and its thickness at the same end is $\frac{4}{5}$ of the breadth; and the back side is made parallel to the face for half the length of the whip: the small end of the whip is square, and at its end is $\frac{1}{18}$ of the length of the whip, or half the breadth at the great end.

From the centre of the shaft B, to the nearest bar 1 of the lattice is $\frac{1}{7}$ of the whip, the remaining space of $\frac{6}{7}$ of the whip is equally divided into 19 spaces; $\frac{1}{3}$ of one of these spaces gives the size of the mortice which must be made square.

To prepare the whip for mortising, strike a gage score at about three quarters of an inch from the face on each side, and the gage score on the leading side, 4, 5, will give the face of all the bars on each side; but on the other side the faces of all the bars will fall deeper than the gage score, according to a certain rule. Which is this—Extend the compasses to any distance at pleasure, so that 6 times that extent may be greater than the breadth of the whip at the seventh bar. Set off these six spaces upon a straight line for a base, at the end of which raise a perpendicular; set off the same six spaces on the perpendicular, and divide the two spaces on the perpendicular which are farthest from the base, each into 6 equal parts, so that these two spaces will contain 13 points. Join each of these 13 points with the end of the base farthest from the perpendicular.

To apply this scale to any given case, set off the breadth of the whip at the last bar (that is the bar at the extremity of the sail,) from the centre of the scale, along the base towards the perpendicular, and at this point raise a perpendicular to cut the oblique line nearest the base; also set off the

breadth at the seventh bar in the same manner, and at this point raise a perpendicular to cut off the thirteenth oblique line. Now, from the point where the first of these two perpendiculars cuts the first oblique line from the base, to the intersection of the second perpendicular with the thirteenth oblique line, there is drawn a line joining the two points of intersection; and perpendiculars being drawn from the points where this joining line cuts the oblique lines to the base, will be the several distances of the face of each bar from the gage line. These distances give a difference, set off for each bar to the seventh, which must be set off for all the rest to the first. The length of the longest bar is $\frac{2}{5}$ of the whip.

Method of weathering the sails.

Draw AB, fig. 97, = the length of the vane, BC its breadth, and BCD the angle of the weather at the extremity of the vane, equal to 20 degrees. With the length of the vane AB, and breadth BC, construct the isosceles triangle ABC from the point B, draw BD perpendicular to CB, then BD is the proper depth of the vane.

Divide the line AB into any number of parts, say five, at these divisions draw the lines 1 E, 2 F, 3 C, and 4 H, parallel to the line BC. Also, from the points of division, 1, 2, 3, and 4, draw the lines 1 I, 2 K, 3 L, and 4 M, perpendicular to 1 E, 2 F, 3 G, &c., all of them equal in length to BD. Join EI, FKGL, and HM, then the angles 1 FI, 2 FK, 3 GL, &c. are the angles of weather, and these divisions of the vane; and if the triangles be conceived to stand perpendicular to the paper, the angles I, KL, M, and D, denoting the vertical angles the hypothenuses of these triangles will give a perfect idea of the weathering of the vane as it recedes from the centre.

HEAT.

As a knowledge of the subject of heat is of the utmost consequence to the practical man, we shall here give a short statement of its principal mechanical properties.

Heat expands bodies, that is, increases their dimensions. Different bodies expand differently by application of the same quantity of heat. With the same degree of heat, solids expand less than liquids, and liquids less than gasses.

On the principle that bodies expand by heat, is constructed the Thermometer. The construction of this instrument is very simple. It consists of a small glass tube with a hollow bulb at one end, and at the other end it is closed. The bulb is filled with mercury, as likewise a part of the tube, the other portion of the tube being entirely deprived of air. When heat is applied to the bulb of the thermometer the mercury expands and rises in the tube, and according to the degree of heat applied to it so will the mercury rise. To the tube there is attached a divided scale, to denote the degrees of heat by the rising of the mercury, which scale is thus formed. The bulb of the thermometer is put into melting ice, and the height of the mercury is marked on the scale, which is called the freezing point, and numbered 32. The bulb is then put into boiling water, and the height of the mercury in the tube is marked upon the scale and numbered 212—this is called the boiling point. The space betwixt these two points on the scale, is divided into 180 equal parts called degrees, and the scale is then extended both above and below these points. This is the scale commonly used in this country, and is known by the name of its

inventor Fahrenheit. But the French and many philosophers in Britain use a thermometer having a scale of much more simple construction, called from the nature of its divisions the *Centigrade scale.* The freezing point, which in Fahrenheit is marked 32, is in the Centigrade marked 0 or zero, and the boiling point in Fahrenheit marked 212, is in the Centigrade marked 100. There are, therefore, in the Centigrade scale 100 degrees betwixt the freezing and boiling points, whereas in the scale of Fahrenheit, there are 180. Wherefore each degree of the Fahrenheit is $= \frac{100}{180}$ or $\frac{10}{18}$ or $\frac{5}{9}$ of a degree of the centigrade; and from this circumstance, and the consideration that the point 32 in Fahrenheit is numbered 0 in the centigrade scale, it is easy to find the value of any degree of temperature on the one scale in terms of the other; thus,

FAHRENHEIT.

$$\frac{(86-32)\times 5}{9} = \frac{270}{9} = 30 = \text{centigrade degree.}$$

$$\frac{(176-32)\times 5}{9} = \frac{720}{9} = 80 = \text{centigrade degree.}$$

And also,

CENTIGRADE.

$$\frac{30\times 9}{5} + 32 = \frac{270}{5} + 32 = 86 = \text{Fahrenheit.}$$

$$\frac{80\times 9}{5} + 32 = \frac{720}{5} + 32 = 144 + 32 = 176 = \text{Fahren.}$$

There are many other particulars regarding the thermometer which it would be inconsistent with the design of these pages to consider: what we have said will be sufficient for the understanding of what is hereafter to follow on the subject of steam, &c.

Before we introduced the subject of the thermometer, we stated the fact of the expansion of bodies by heat. Bars of the following substances whose length at a temperature of 32 was

1, were heated to 212 Fahrenheit, and expanded so as to become,

Cast iron,	1·00110940
Steel,	1·00118990
Copper,	1·00191880
Brass,	1·00188971

This is the expansion in length; the expansion in length, breadth, and thickness, will be found by multiplying the above numbers by 3.

The effects of different degrees of heat on different bodies, according to Fahrenheit's scale.

Cast iron thoroughly melted,	20577
Cast iron begins to melt,	17977
Greatest heat of a common smith's forge,	17327
Flint glass furnace, strongest heat,	15897
Welding heat of iron, (greatest,)	13427
Swedish copper melts,	4587
Brass melts,	3807
Iron red hot in the twilight,	884
Heat of a common fire,.	790
Iron bright red in the dark,	752
Zinc melts,	700
Mercury boils,	672
Lead melts,	594
The surface of polished steel becomes uniformly deep blue,	580
——— becomes a pale straw colour,	460
Tin melts,	442
A mixture of 3 tin and 2 lead melts,	332

Heat passes through different bodies with very different degrees of velocity, and according to the rapidity or slowness with which heat passes through any body, it is said to be a good or a bad conductor of heat.

The conducting power of copper being 1, that of brass will be 1, iron, 1·1, tin, 1·7, lead, 2·5. The densest bodies

are generally the best conductors of heat; but this is not universal, as platina, one of the densest of all metals, is one of the worst conductors. Earthy substances are much inferior to metals in their conducting power, and the worst conductors of all are the coverings of animals.

When heated bodies are exposed to the air they lose portions of their heat by projection in right lines into space from all parts of their surface. This is called the radiation of heat.

Bodies which radiate heat best have the power of absorbing it in the same proportion, and the least power of reflecting it; hence, in leading steam through a room, it would be absurd to use black pipes, because, in that case, much of the heat would escape by radiation before the steam would be carried to the place where it was to be used. If the steam is used to heat the apartment, black pipes are the best. Vessels intended to receive heat should be black.

The comparative quantities of heat existing in different bodies may be ascertained by marking the time which equal quantities of them require to cool a certain number of degrees, reckoning their capacities for heat to be as these times estimated by the volume; or, if divided by the specific gravity of the substance, by the weight.

Large quantities of heat must enter into bodies, and be concealed to enable them to pass from the solid to the fluid state, or from the fluid state to that of vapour. Thus the quantity of heat necessary to convert any give weight of ice into water, would raise the same weight of water 140 degrees of Fah. This quantity of heat is not sensible, but is, as it were, kept hid or *latent;* nor can it be detected by the touch, or by application of the thermometer.

Every addition of heat applied to water in a fluid state, raises the temperature until it arrives at the boiling point; but however violently the fluid may boil, it does not become hotter, nor does the steam that arises from it indicate a

greater degree of heat than the water; hence, a large proportion of the heat must enter into the steam and become latent. The quantity of heat that becomes latent in the steam was found, by Dr Black, to be 810, of Fah.

Under the common pressure of the atmosphere at the surface of the earth, (= 15 lbs. to the square inch,) water cannot be raised above a temperature of 212 Fah.; but when exposed to greater pressure, by being confined in a vessel, the water may be raised to a much higher degree of heat, and if, in this state of confinement, the heat applied be insufficient to cause the water to boil: if the vessel should be open, steam will rush out, and the water which remains will fall in temperature to 212. On the contrary, water boils at very low temperatures when the pressure is diminished; as in an exhausted receiver, or at the tops of mountains.

When the temperature of steam is reduced, it assumes again the fluid form, and the quantity of latent heat set free by steam in passing to the state of water, has been found, by Mr Watt, to be 945 degrees. He also found that a cubic inch of water may be converted into a cubic foot of steam; and that when this steam is condensed, by injecting cold water the latent heat which the steam gives out in passing to the fluid state, would be sufficient to heat 6 cubic inches of water to the temperature of 212, or the boiling point. It is generally considered that steam raised from boiling water occupies 18 hundred times as much space as the water did from which it was raised, and instead of making the latent heat of steam 810, as Dr Black found it, more correct experiments show it to be 1000, at the common pressures of the atmosphere; but the latent heat of steam is inversely proportional to the degree of pressure under which it is produced; that is, the latent heat is greatest where the pressure is least, and least where the pressure is greatest.

It has lately been discovered that the sensible heat and latent heat of steam at any one temperature added together,

give a sum which is constant; that is to say, which is the sum of the sensible and latent heat of any other temperature, or under any other pressure. Now, the sensible heat of steam at the ordinary pressure of the atmosphere is $212 - 32 = 180$; and the latent heat has been found to be 1000, their sum is 1180, which is the constant sum of the latent and sensible heats of steam under any other pressure. Thus, at the temperature of 248, where the elastic force of the steam is equal to two atmospheres, or a pressure of 30 lbs. on the square inch, the sensible heat will be $248 - 32 = 216$, wherefore the latent heat is $1180 - 216 = 964$, and so of the other temperatures.

It has also been found that while the elasticity of steam increases in geometrical progression, with a ratio of 2, the latent heat diminishes with a ratio of 1·0306, differing not very materially from unit.

Many experiments have been made to ascertain the elastic force of steam of various temperatures. The most valuable of which are those recently made by the French academicians, the results of which are here given in a tabular form; and the practical man will duly estimate the value of this gift of science.

TABLE OF THE ELASTICITY OF STEAM.

Elasticity of steam, the pres. of the atmosphere being 1.	Corresponding temp. in deg. of Fahrenheit.	Elasticity of steam, the pres. of the atmosphere being 1.	Corresponding temp. in deg. of Fahrenheit.
1	212°	13	380·66
$1\frac{1}{2}$	234	14	386·94
2	250·5	15	392·86
$2\frac{1}{2}$	263·8	16	398·48
3	275·2	17	403·83
$3\frac{1}{2}$	285	18	408·92
4	293·7	19	413·78
$4\frac{1}{2}$	300·3	20	418·46
5	307·5	21	422·96
$5\frac{1}{2}$	314·24	22	427·28
6	320·36	23	431·42
$6\frac{1}{2}$	326·26	24	435·56
7	331·7		
$7\frac{1}{2}$	336·86	25	439·34
8	341·78	30	457·16
9	350·78	35	472·73
10	358·78	40	486·59
11	366·85	45	499·14
12	374	50	510·6

OF THE STEAM ENGINE.

It is not consistent with the plan of this book, that we should enter into minute details, as to all the modifications and departments of the steam engine; a subject which would of itself occupy a large volume. We shall, however, attempt to explain the leading principles on which this invaluable machine operates, so that the mode of calculating its effects may be the more clearly comprehended.

1. Let there be a sucking pipe with a valve opening upwards at the top, communicating with a close vessel of water, not more than thirty-three feet above the level of the reservoir, and the steam of boiling water be thrown on the surface

of the water in the vessel, it will force it to a height as much greater than thirty-three feet as the elastic force of the steam is greater than that of air; and if the steam be condensed by the injection of cold water, and a vacuum thus formed, the vessel will be filled from the reservoir by the pressure of the atmosphere; and the steam being admitted as before, this water will also be forced up; and so on successively.

Such is the principle of the first steam-engine, said by the English to be invented by the Marquis of Worcester; while the French ascribe it to Papin: though we believe the fact is that Brancas, an Italian, applied the force of steam ejected from a large æolopile as an impelling power for a stamping-engine so early as 1629. Brancas's was, in fact, an immense blow-pipe, turning a wheel. The hint so obscurely exhibited in the Marquis of Worcester's century of inventions was carried into effect by Captain Savery.

2. If the steam be admitted into the bottom of a hollow cylinder, to which a solid piston is adapted, the piston will be forced upwards by the difference between the elastic forces of steam and common air; and the steam being then condensed, the piston will descend by the pressure of the atmosphere, and so on successively. This is the principle of the steam engine first contrived by Messrs Newcomen and Cawley, of Dartmouth. This is sometimes called the atmospherical engine, and is commonly a forcing pump, having its rod fixed to one end of a lever, which is worked by the weight of the atmosphere upon a piston at the other end, a temporary vacuum being made below it by suddenly condensing the steam, that had been admitted into the cylinder in which this piston works, by a jet of cold water thrown into it. A partial vacuum being thus made, the weight of the atmosphere presses down the piston, and raises the other end of the straight lever, together with the water from the well. Then immediately a hole is uncovered in the bottom of the cylinder, by which a fresh quantity of hot steam rushes in

from a boiler of water below it, which proving a counterbalance for the atmosphere above the piston, the weight of the pump-rods, at the other end of the lever, carries that end down, and raises the piston of the steam-cylinder. The steam hole is then immediately shut, and a cock opened for injecting the cold water into the cylinder of steam, which condenses it to water again, and thus making a vacuum below the piston, the atmosphere again presses it down and raises the pump-rods, as before; and so on continually.

3. When the cylinder is full of steam, if a valve be opened, by which the steam is allowed to escape into another vessel, where a jet of cold water is introduced; the condensation is much more complete, and the heat of the cylinder being preserved, the steam possesses its full elasticity.

This improvement was made by Mr Watt, and completely changed the character of the steam engine. In the old engines the power was diminished to half its real value, so that the moving force, instead of reaching 15 lbs. on each square inch of the area of the piston, was reduced to about 8 lbs. In this engine of Mr Watt's the moving force is not less than 12 lbs. upon each square inch of the piston.

4. A farther improvement has been made on this engine, by injecting the steam into the cylinder, alternately above and below the piston, so that the whole motion is produced by the elasticity of the steam, and has no dependence on the weight of the atmosphere.

This improvement is also due to Mr Watt, and could not have been made without the previous contrivance of condensing the steam in a separate vessel. It is particularly accommodated to the production of a rotary motion by means of a steam engine. Three years before Mr Watt introduced this improvement, viz. in 1778, Mr Washborough of Bristol, took out his patent for converting a reciprocating into a rotatory motion; and in 1781, Mr J. Steed effected the same thing, for the first time, by means of what is now called a

crank. From that time Hornblower, Cartwright, Murray, Bramah, Trevithick, Maudslay, Woolf, and others, have, in rapid succession, introduced a series of improvements which have rendered steam engines as efficacious and perfect as can well be conceived.

5. Another improvement due to Mr Watt, is that of the expansion engine, invented about 1769. The principle of this invention, as Mr Partington correctly remarks, consists in shutting off the farther entrance of steam from the boiler when the piston has been pressed down in the cylinder, for a certain proportion of its total descent, leaving the remainder to be accomplished by the expansive force of the steam already produced.* To regulate the time of closing the valve, and as such the precise amount of steam admitted, Mr Watt employed a plug-frame with moveable pins, which may be so placed that the steam valve will shut when the piston has descended one-half, one-third, one-fourth, &c.. By the application of this principle, the piston is made to descend uniformly, the pressure on it continually diminishing as the steam becomes more and more rare, and the accelerating force is consequently diminished.

6. The principle of the high-pressure steam engine depends also on the power of steam to expand itself very considerably beyond its original bulk, by the addition of a given quantity of caloric, thus acquiring a considerable elastic force (equivalent to from 40 to 60 lbs. on each square inch) which, in this case, is employed to give motion to a piston. One of the greatest advantages attendant on employing the repellent force of steam, as in this form of the engine, consists in an evident saving of the water usually employed in condensation; and this, in locomotive engines for propelling carriages, is an object of considerable importance. The first description of an engine of this kind, which we have

* Gregory's Mathematics for practical men.

met with, is in Leupold's Theatre of Machines, published in Germany, in 1724. The apparatus consists of two cylinders placed at a moderate distance asunder; each of them provided with a piston made to fit air-tight, and connected with a forcing pump. When steam of considerable elasticity is admitted at the bottom of the first cylinder, it is forced upwards, carrying with it the lever of the pump; at the same time that the steam or air is expelled from the other. On this operation being repeated, or rather, reversed, the steam is allowed to enter the second cylinder, which is also connected with the boiler, while the steam in the first cylinder is allowed to escape into the air. Thus, it may be remarked, that the process of condensation forms no part of the principle of the high pressure engine; and that even the expansion of gunpowder might be employed to produce a similar effect.

7. Mr Woolf made the discovery, that a quantity of steam having the force of 5, 6, 7, or more pounds on every square inch of the boiler, may be allowed to expand itself to an equal number of times its own volume, when it would still have a pressure equal to that of the atmosphere, provided that the cylinder in which the expansion takes place have the same temperature as the stream possessed before it began to increase.

The most economical mode of employing this principle consists in the application of two cylinders and pistons of unequal size to a high pressure boiler; the smaller of which should have a communication both at its top and bottom with the steam vessel. A communication being also formed between the top of the smaller cylinder, and the bottom of the larger cylinder; and *vice versa*. When the engine is set to work, steam of a high temperature is admitted from the boiler to act by its elastic force on one side of the smaller piston, while the steam which had last moved it has a communication with the larger or condensing cylinder. If both

pistons be placed at the tops of their respective cylinders, and steam of a pressure equal to 40 lbs. on the square inch be admitted, the smaller piston will be pressed down, while the steam below it, instead of being allowed to escape into the atmosphere, or pass into the condensing vessel, as in the common engine, is made to enter the larger cylinder above its piston, which will make its downward stroke at the same time as that in the smaller cylinder; and during this process, the steam which last filled the larger cylinder, will be passing into the condenser to form a vacuum during the downward stroke.

To perform the upward stroke it is merely necessary to reverse the action of the respective cylinders; and it will be effected by the pressure of the steam in the top of the small cylinder, acting beneath the piston in the great cylinder; thus alternately admitting the steam to the different sides of the smaller piston, while the steam last admitted into the smaller cylinder passes regularly to the different sides of the larger piston, the communication between the condenser and steam boiler being reversed at each stroke.

Mr Partington states that a double cylinder expansion engine of this kind was constructed for Wheal Vor mine in Cornwall, in 1815. In this, the great cylinder is 53 inches in diameter, and has a nine feet stroke; the small cylinder being in content about one-fifth of the great one. The engine works 6 pumps, which at every stroke raise a load of water of 37,982 lbs. weight, 7½ feet high. This produces a pressure of 14·1 lbs. per square inch on the surface of the great piston, while its average performance has been estimated at 46,000,000 lbs. raised one foot high with each bushel of coals.

From what has been said, it will not be difficult for the attentive reader to follow our remarks on the mode of calculating the effects of this wonderful machine.

It is clear, that the power of the steam engine will depend,

in the first place, upon the energy of the steam—steam two atmospheres will, other things being equal, produ double the effect of steam of one atmosphere. It will al appear that, the force of the steam remaining the same, th power of the engine will depend on the extent of surfac acted upon, that is on the area of the piston. These two cir cumstances remaining the same, it is also evident, that the power of the engine will depend on the distance which the piston moves, that is the length of stroke.

For the sake of illustration, let us suppose that steam is admitted into the cylinder, so as to press down the piston with the force of one hundred pounds, and that the length of the stroke is five feet; and suppose that the end of the piston rod is attached to a beam whose fulcrum is in the centre; and that to the other end of the beam, there is attached a weight of one hundred pounds, there being no friction. By the descent of the piston the weight at the end of the beam will be raised 5 feet; therefore it follows, that 100 lbs. raised 5 feet during one descent of the piston, will express the mechanical effect of the engine. If we suppose the area of the piston double of what it was before, other things being the same, the engine would raise 200 lbs. through the same space of 5 feet in the same time: and the same effect would evidently ensue if we supposed the area of the piston to remain as it was at first, but the force of the steam to be doubled,—for then, the engine would raise 200 lbs. 5 feet high in one descent of the piston. If the area of the piston and force of steam be the same as at first, but the length of stroke doubled, then the mechanical effect of the engine will be 100 lbs. raised 10 feet high, during one descent of the piston; and if the descents be performed in the same time, this engine will be double the power of the first.

Let us proceed now to actual cases. In the common low pressure steam engine of Watt, steam is admitted into the cylinder whose elastic force is somewhere about that of the

atmosphere, which we have all along supposed to be 15 lbs. to the square inch; but friction and imperfect vacuums tend to diminish this pressure, and the effective pressure may therefore be taken four-fifths of this. If the pressure of the steam is diminished by its one fifth part, which is 3 lbs. to the square inch, then will the effective pressure be 12 lbs. to the square inch. The working pressure is generally reckoned at 10 lbs. to the circular inch, and Smeaton only makes it 7 lbs. The effective pressure we have taken is between these extremes being equivalent to 9·42 lbs. to the circular inch.

If we now suppose a cylinder whose diameter is 24 inches, the area of this cylinder, and consequently the area of the piston will be,

$$24^2 \times \cdot 7854 = 452 \cdot 29 =$$

the area of the piston in square inches. Steam is admitted into the cylinder, as we have supposed, having an effective pressure on the piston of 12 lbs. to the square inch; therefore, $452 \cdot 29 \times 12 = 5427 \cdot 48$ lbs., the whole force with which the piston is pressed. If we now suppose that the length of the stroke is five feet, and the engine makes 44 single or 22 double strokes in a minute, then the piston will move through a space of $22 \times 5 \times 2 = 220$ feet in a minute; and from what has been said before, it will not be difficult to see, that the power of the engine will be equivalent to a weight of 5427 lbs. raised through 220 feet in a minute.

This is the most certain measure of the power of a steam engine. It is usual, however, to estimate the effect as equivalent to the power of so many horses. This method, however simple and natural it may appear, is yet, from differences of opinion as to the power of a horse, not very accurate; and its employment in calculation can only be accounted for on the ground, that when steam engines were first employed to drive machinery, they were substituted instead of horses;

and it became thus necessary to estimate what size of steam engine would give a power equal to so many horses.

There are various opinions as to the power of a horse. According to Smeaton, a horse will raise 22916 lbs. one feet high in a minute. Desaguliers makes the number 27500, and Watt makes it larger still, that is 33000. There is reason to believe that even this number is too small, and that we may add at least 11000 to it, which gives 44000 lbs. raised one foot high per minute.

Now, in the case above, we found that the engine of 24 inch cylinder, would raise 5427 lbs. through the space of 220 feet in one minute; and it is easily seen that it could raise 220×5427 lbs. through one foot in the same time, therefore $220 \times 5427 = 1193940$ lbs. raised though one foot in one minute, is the effective power of the engine; and from these considerations it will be easy to find the power according to the different estimates of a horse's power. For,

$$\frac{1193940}{22916} = 52 \text{ horses' power,}$$

according to Smeaton.

$$\frac{1193940}{27500} = 43 \text{ horses' power}$$

according to Desaguliers.

$$\frac{1193940}{33000} = 36 \text{ horses' power}$$

according to Watt.

$$\frac{1193940}{44000} = 27 \text{ horses' power}$$

according to the usual estimate.

The reader will have no difficulty in forming a general rule for estimating the power of a steam engine. (The effective pressure on each square inch $\times$ the area of piston in square inches $\times$ length of stroke $\times$ feet $\times$ number of strokes per minute) $\div$ 44000 = the number of horses' power of the engine.

What is the power of a low pressure engine, whose cylinder is 30 inches diameter, length of stroke 6 feet, making 16 double strokes in the minute.

Note.—An easy rule to find the area of the piston in square inches, is this,

$$\frac{\text{The diameter} \times \text{circumference}}{4} = \text{area.}$$

Here we have,

$$\frac{(30 \times 3{\cdot}1416) \times 30}{4} = \frac{2828{\cdot}44}{4} = 707{\cdot}11$$

equal the area of the piston in square inches; and 12 the effective pressure, 6 the length of stroke, 16 the number of double strokes in a minute.

$$\frac{707{\cdot}11 \times 12 \times 6 \times 16 \times 2}{44000} = \frac{1629161{\cdot}44}{44000} = 37$$

horses' power.

If the cylinder of a high pressure steam engine, has a cylinder of 5 inches diameter, with a twelve inch stroke, making 32 double strokes in a minute; steam being admitted of an elastic force equivalent to 7 atmospheres on the inside of the cylinder. Its effective pressure will be $7 \times 15 = 105$ lbs. to the square inch without friction; but allowing one fifth for friction, the effective pressure will be $105 - 21 = 84$ lbs. to the square inch.

here, $\frac{5 \times 3{\cdot}1416 \times 5}{4} = 19{\cdot}63$ the area of cylinder,

hence, $\frac{19{\cdot}63 \times 84 \times 1 \times 32 \times 2}{44000} = \frac{105520{\cdot}88}{44000} = 2$

horses' power.

We might simplify this rule still farther, on the consideration, that the divisor 44000 may be viewed as the denominator of a fraction whose numerator is one, and by converting this into a decimal, and multiplying by it, we might avoid the necessity of division.

$\frac{1}{44000} = \cdot 0000227$, hence we may devise the rule.

Effective pressure of steam $\times$ area of piston in square inches $\times$ length of stroke in feet $\times$ number of strokes per minute $\times$ 227; cutting off seven places as decimals,—the answer is the horses' power of the engine.

This is for a single stroke engine—for a double stroke engine the multiplier is $227 \times 2 = 454$.

If the cylinder be 42 inches diameter, and the piston moves 210 feet per minute, then the engine being low pressure, we have,

$$\text{area of cylinder equal } 1385\cdot19\text{; hence } 227 \times 1385\cdot19 \times 210 \times 12 = 792275400:$$

and the seven figures cut off as decimals, leave 79 horses' power.

These are at best but approximations, and for safety it might be advisable that a lower number than 12 should be employed, as the effective pressure of the steam; the number 10 may be used as being easily managed, and coming near the truth; and thus the above rule may be simplified by neglecting the pressure of the steam, and cutting off six places for decimals instead of seven, as there is reason to believe that the above results will answer only ponies instead of strong horses.

When we come to speak of the effects of machines, we will state other particulars as to the relative power of the steam engine. In the meantime, we add only a few promiscuous remarks on the various parts of the common steam engine.

The stroke of an engine is commonly reckoned equal to one complete revolution of the crank shaft, and therefore double the length of the cylinder; and it is to be observed, that there is a certain relation between the velocity of the piston, and the length of stroke at which the power of the engine will be a maximum.

It has been stated by the first engineer of the day, that to ascertain the velocity of the piston when the engine performs at its maximum, we may employ the rule,

$$120 \times \text{length of stroke}^{\frac{1}{2}} = \text{velocity.}$$

If an engine has a two feet stroke, then,

$$120 \times 2^{\frac{1}{2}} = 120 \times 1{\cdot}4142 = 169{\cdot}704.$$

or we may say 170, as the velocity of the piston per minute in feet; wherefore as the engine has a single stroke of 2 feet we have,

$$\frac{170}{4} = 42\tfrac{1}{2} \text{ strokes in the minute.}$$

If an engine have a four feet stroke, then we have,

$$120 \times 4^{\frac{1}{2}} = 120 \times 2 = 240 =$$

the velocity of the piston per minute; and,

$$\frac{240}{8} = 30, \text{ equal the number of strokes per}$$

minute.

The safety valves of most of the steam engines in this part of the country, are generally loaded with a weight of from 3 to 4 lbs. to the square inch of their area; let us take 3½ lbs. in the present instance. The temperature of steam necessary to balance this pressure, is, according to the best experiments, 223 degrees of Fahrenheit's thermometer. But besides this sensible heat, there is a quantity of latent heat not indicated by the thermometer, and which can only be detected when the steam passes, by condensation, into the fluid state; as the latent heat is then given out. Now, if the latent heat of the steam at the above temperature, be found on the principle stated in our remarks on heat, that the sensible and latent heats of steam at all temperatures, when added together, made a constant quantity; we will find that the latent heat of steam at this temperature is 989, which, as has been said above, is not indicated by the thermometer. The real quantity of heat then in the steam is, $223 + 989 = 1212$ degrees. We will not be far from the

truth in supposing, that one cubic foot of this steam will, when condensed into water, measure one cubic inch; and the steam is supposed to be condensed by the injection of cold water. Now it is evident, that the temperature of the water formed by the condensation of the steam, will be somewhere between the temperature of the cold water and the boiling point. Say that the temperature of the injected water is 50 degrees, and that the temperature of the water arising from the condensation of the steam is 100. We must deduct the 100 degrees from the heat of the uncondensed steam, that is, 1212 — 100 = 1112, which is left to be communicated to the injection water; and since each cubic inch of the cold water requires 50 of heat to raise it to the temperature of the water found after the condensation of the steam, therefore,

$$\frac{1112}{50} = 22\tfrac{3}{15} \text{ cubic inches}$$

of water necessary to condense one cubic foot of steam to the temperature of 100, the injected water being 50.

From these considerations, may be derived a rule for determining the quantity of water necessary to condense any quantity of steam, at any given temperature.

$$\frac{\text{Total heat of the steam} - \text{temperature of warm water}}{\text{temp. of warm water} - \text{temp. of cold water}} \times$$

quantity of steam in cubic feet = the quantity of cold water in cubic inches necessary to produce the effect.

Let us illustrate this by an example—what quantity of cold water will it require of the temperature of 60, to condense 8 cubic feet of steam, of the temperature of 223, to water at 90. The whole heat is as before, 989 + 223 = 1212, wherefore by the rule,

$$\frac{1212 - 90}{90 - 60} \times 8 = 299{\cdot}2 \text{ cubic inches} =$$

$$\frac{299 \cdot 2}{1728} = \cdot 17 \text{ of a cubic foot of water.}$$

From this it will be easy to determine how much water must be discharged by the pump which feeds the condenser, in order that a proper vacuum may be formed.

It would appear from observation, that the engines commonly in use require betwixt 7 and 8 gallons of cold water per minute for each horse power. If the water is returned as it is in some engines, then a greater quantity will be necessary. Now, in the usual construction of engines, the pump rod which supplies the condenser with cold water, is fixed half way between the end of the beam and the centre; hence, the length of its stroke is one half that of the piston in the large cylinder: therefore, if there be a 40 horse power engine, the length of whose stroke is 6 feet, the length of the stroke of the pump will be 3 feet.

Now a wine gallon occupies a space of 231 cubic inches, and $7\frac{1}{2}$ gallons will occupy a space of $231 \times 7 \cdot 5 = 1733 \cdot 5$ cubic inches; and as the engine is 40 horses' power, there must be discharged in one minute,

$$1733 \cdot 5 \times 40 = 45340 \text{ cubic inches,}$$

and if the engine makes 30 strokes per minute, then,

$$\frac{45340}{30} = 1511 \text{ cubic inches}$$

discharged at one stroke: but the stroke is 3 feet long, and it remains only to find what must be the diameter of a pump's bore, whose length is 36 inches, so that its capacity shall be 1511; hence we find that,

$$\frac{1511}{36} = 41 \text{ inches,}$$

nearly equal the area of the pump's bore; now the area of circles are to each other as the squares of their diameters, by the principles of geometry. And it may be found that the area of a circle whose diameter is 9, is 63·6; therefore,

$$63 \cdot 6 : 41 :: 9^2 : 52$$

[illegible] will be the diameter of the pump [illegible] = [illegible] or [illegible] inches.

[illegible] the condensing [illegible] engine [illegible] power, the capacity of the [illegible] given before will [illegible] required; from which [illegible] the diameter of the pump.

The capacity [illegible] pump should be one fourth of the [illegible] cylinder. The diameter of the valves in the [illegible] diameter of cylinder as 5 to [illegible]. The [illegible] each stroke by the hot water [illegible] the [illegible]-hundredth part of the [illegible] cylinder. The working beam should be [illegible] length of the stroke. The length of connecting [illegible] length of stroke. The crank's length [illegible] length of stroke. In the parallel [illegible] are equal, and about the [illegible] length of the beam between the gland[illegible] [illegible] length between the centres, and [illegible] less than half the length of [illegible]. [illegible] and fly wheel will be [illegible] book, where they may be [illegible]. We may here, however, give the [illegible] rule for the determination of the weight of the fly wheel.

$$\frac{\text{Steam power of engine} \times 2000}{\text{[illegible] of circumference of wheel in feet per second}^2} = \text{the weight of the wheel in cwts.}$$

[illegible] the diameter of the fly of a 30 horse power engine is 20 feet, and makes 18 revolutions per minute, then,

$$20 \times 3.1416 = 62.832 =$$

circumference in feet, and $62.832 \times 18 = 1130.97$ feet the space which the circumference moves through in one minute: hence

$$\frac{1128 \cdot 97}{60} = 18 \cdot 81 \text{ feet per second;}$$

hence, $\frac{30 \times 2000}{18 \cdot 81^2} = \frac{6000}{353 \cdot 8} = 169$ cwts.

= 8 tons 9 cwts. the weight of the fly.

We have before spoken of the governor while treating of central forces and rotation, and it remains for us here only to observe, that the governor performs in one minute half as many revolutions as a pendulum, whose length is the perpendicular distance between the plane in which the balls move and the centre of suspension. Thus, if the distance between the point of suspension and the plane in which the balls move be 28 inches:

$\left(\frac{39 \cdot 1386}{28}\right)^{\frac{1}{2}} = 1 \cdot 145$ vibrations in a second from the nature of the pendulum; hence,

$\frac{1 \cdot 145}{2} = 0 \cdot 572$ the revolutions of the governor in a second, or $0 \cdot 572 \times 60 = 33 \cdot 32$ in one minute.

It will be necessary to consider the strength of the vessels in which the steam is either formed or contained; and this may be done on the same principle that we calculated the strength of the cylinder of the Hydrostatic press. The rule is this;

(The pressure per square inch $\times$ the radius of the cylinder) $\div$ (the cohesive power of the metal per square inch — pressure per square inch) = the thickness of metal in inches.

Now if steam of 6 atmospheres be contained in a cylindrical vessel of cast iron 3 feet diameter, we have by the rule,

$6 \times 15 = 90 =$ the pressure of steam 18 inches, the radius of cylinder, and 18000 the cohesive strength of cast iron, therefore, by the rule.

$$\frac{90 \times 18}{18000 - 90} = \frac{1620}{17910} = \frac{162}{171} \text{ parts of an inch.}$$

Many rules have been given for the quantity of fuel ne-

cessary for the production of steam, but they cannot be depended on, so many circumstances must be taken under consideration--the quality of the material used for fuel, and the mode of constructing the fire place.

It has been found that 3 cwt. of Newcastle coals are equivalent to 4 hundred weight of Glasgow coals, or 9 cwt. of wood or 7 cwt. of culm. A chaldron of coals in London, contains 36 bushels, and weighs 3136 lbs. or nearly 1 ton 8 cwt.

It would appear, that in the common low pressure steam engines, the consumpt of coal per hour for 1 horse power, is about 16 lbs., of wood 56 lbs., and of culm 35 lbs. These statements are given somewhat large, but by proper regulation much less fuel might serve.

In the boiler there are certain proportions generally observed. The width, depth, and length are as the numbers 1, 1·1, 2·5. So that if the width be 5 feet, then the depth will be $1 \cdot 1 \times 5 = 5$ feet 6 inches, and the length $5 \times 2 \cdot 5 = 12$ feet 6 inches; and the whole content of the boiler will be,

$$5 \times 5 \cdot 5 \times 12 \cdot 5 = 343 \cdot 75 \text{ cubic feet.}$$

Now Boulton and Watt allow 25 cubic feet of space in the boiler for each horse power; and according to this estimate,

$\frac{343 \cdot 75}{25} =$ 13 and a fraction, the number of horses' power of this engine for which this boiler would be fitted. Some, instead of computing the size of boiler in this way, allow 5 square feet of surface of water for each horse's power; but in all cases, it is common, to make the boiler of a size fitted for an engine of at least 2 horses' power more than that to which it is applied.

TABLE I.

Maximum power of the steam of a cubic foot of water, in high pressure steam-engines.

Temperature of steam.	Total force of steam in inches of mercury.	Force of steam in lbs. per sq. inch above the atmos.	Maximum mechanical power of the steam of a cubic foot of water, in lbs. raised one foot high. When working at full pressure.	Maximum mechanical power of the steam of a cubic foot of water, in lbs. raised one foot high. Acting expansively.	Proportion of the stroke to cut off the steam to obtain the maximum by expansion.	Pounds of Newcastle coal to convert 1 cubic foot of water into steam.
	inches.	lbs.	lbs.	lbs.		lbs.
220°	35	2·5	negative.	negative.		8·5
234½	45	7·4	287,000			8·67
251	60	14·8	864,000	985,000	$\frac{3}{4}$	8·87
275	90	29·7	1,495,000	1,927,000	$\frac{7}{12}$	9·16
292·8	120	44·5	1,830,000	2,540,000	$\frac{1}{2}$	9·37
307·7	150	59·3	2,054,000	2,988,000	$\frac{9}{20}$	9·55
320·2	180	74·2	2,202,000	3,326,000	$\frac{6}{12}$	9·7
343·6	240	104	2,444,000	3,832,000	$\frac{3}{8}$	9·98

TABLE II.

Maximum power of the steam of a cubic foot of water, in a condensing steam-engine.

Temperature of steam.	Total force of steam in inches of mercury.	Force of steam in lbs. per sq. inch above the atmos.	Maximum mechanical power of the the steam of a cubic foot of water in lbs. raised one foot high. When working at full pressure.	Maximum mechanical power of the the steam of a cubic foot of water in lbs. raised one foot high. Acting expansively.	Proportion of the stroke to cut off the steam to obtain the maximum expansively.	Pounds of Newcastle coal to convert 1 cubic foot of water into steam.
	inches.	lbs.	lbs.	lbs.		lbs.
220°	35	2·5	2,134,000	3,350,000	$\frac{4}{11}$	8·5
234·5	45	7·4	2,230,000	3,636,000	$\frac{15}{44}$	8·67
251	60	14·8	2,366,000	3,961,000	$\frac{7}{22}$	8·87
275	90	29·7		4,379,000	$\frac{10}{34}$	9·16
292·8	120	44·5		4,590,000	7	9·37
307·7	150	59·3		4,819,000	$\frac{10}{36}$	9·55
320·2	180	74·2		4,932,000	$\frac{3}{11}$	9·7
343·6	240	104·		5,162,000	$\frac{4}{15}$	9·98

RAILWAYS.

It has been deduced from very extensive experiments on the Liverpool and Manchester railways, that the effective power of a locomotive engine is about ·3 of the pressure of the steam on the piston, on the calculated power of the engine being 1. In one case, for instance, a cylinder 21 inches diameter was used, the elasticity of steam in the boiler was 30 lbs. to the square inch, above the pressure of the atmosphere. The length of the rail, which was inclined, was 3165 feet, and the height 24 feet. The time of drawing 6 loaded waggons, each weighing 9010 lbs. up the rail, was 570 seconds, during which time the engine made 444 single strokes, each 5 feet long. Now,

$21^2 \times \cdot7854 = 346\cdot36 =$ the area of the piston in square inches, wherefore, $346\cdot36 \times 30 = 10390$ lbs. $=$ the pressure of steam upon the piston, whose stroke was 5 feet, and number of strokes in the given time 444; hence $444 \times 5 = 2220$ feet $=$ the space through which the power 10390 has traversed; therefore, $10390 \times 2220 = 23067576$ lbs. $=$ the impelling power of the engine. Now, it was found that the actual weight including resistance moved, was 7124415 lbs.; then,

$\frac{7124415}{23067576}$ which will give the effect about 30·9 per cent, but the foregoing number may be taken as a safe medium, that is 30 per cent or ·3.

The amount of retardation, arising from steam carriages moving on railways, has been estimated thus;

Loaded carriages weighing altogether 8522 lbs. the friction amounted to 50 lbs., or the $\frac{1}{170}$ part of the weight. In empty carriages weighing 2586 lbs., the friction amounted to 10 lbs. or the $\frac{1}{258}$ part of the weight; and the friction

may be regarded as a constant retarding force. Wrought iron rails seem from a multitude of experiments to be much better than cast iron rail, as they are more durable and cause less friction.

The Rocket, a steam carriage on the Liverpool and Manchester railway was tried. It weighed 4 tons and 5 cwt., to it there was attached a tender with water and coals, weighing 3 tons, 2 cwt. 0 quar. 2 lbs.; and two carriages loaded with stones, weighing 9 tons, 10 cwt. 3 qr. 26 lbs. making in all 17 tons. At full speed she moved at the rate of 30 miles in 2 hours, 6 minutes, 9 seconds, or 14¼ per hour; at the end of stage about 6 miles, and the greatest velocity was 29½ miles per hour. The quantity of water used 92·6 cubic feet, and it required $11\frac{7}{10}$ lbs. of coak for each cubic foot of steam.

In the Rocket the boiler is cylindrical, with flat ends 6 feet long, and 3 feet 4 inches in diameter. To one end of the boiler there is attached a square box as a furnace, 3 feet long by 2 feet broad, and about 3 feet deep—at the bottom of this box five bars are placed, and the box is entirely surrounded with a casting, except at the bottom and the side next the boiler. Betwixt the casting and the box there is left a space of about 3 inches, which is kept constantly filled with water. The upper half of the boiler is used as a reservoir for steam; the under half being kept filled with water, and through this part copper tubes reach from one end to the other of the boiler, being open to the fire box at one end, to the chimney at the other; these tubes are 25 in number, each being 3 inches in diameter. The cylinders were each 8 inches in diameter, and one was at each side of the boiler; the piston had a stroke of 16½ inches. The diameter of the large wheels was 4 feet 8½ inches. The area of the surface of water, exposed to the radiant heat of the fire, was 20 square feet, being that surrounding the fire box or furnace; and the surface exposed to the heated air or flame

from the furnace, or what may be called communicati
heat, is 117·8 square feet.

The average velocity of the Rocket may be stated at 1·
miles per hour, and during one hour she evaporates 18·24
cubic feet of steam, with a consumpt of about 17·7 lbs. of
coak for each cubic foot of water.

An empirical rule has been given for the ascertaining of
the quantity of fuel necessary for steam carriages which
may be useful.

$$\frac{\text{The weight of the load} \times 51 \cdot 55 + \text{weight of carriages}}{898} =$$

the quantity of coals required to carry one mile,—but a near
approximation to the truth may be to allow 2 lbs. for every
ton for one mile.

TABLE.

Showing the effects of a force of traction of 100 lbs. at different velocities, on canals, rail-roads, and turnpike-roads.*

Velocity of motion.		Load moved by a power of 100 lbs.					
		On a Canal.		On a level Railway.		On a level Turnpike Road.	
Miles per hour.	Feet per second.	Total mass moved.	Useful effect.	Total mass moved.	Useful effect.	Total mass moved.	Useful effect.
		lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
2½	3·66	55,500	39,400	14,400	10,800	1,800	1,350
3	4·40	38,542	27,361	14,400	10,800	1,800	1,350
3½	5·13	28,316	20,100	14,400	10,800	1,800	1,350
4	5·86	21,680	15,390	14,400	10,800	1,800	1,350
5	7·33	13,875	9,850	14,400	10,800	1,800	1,350
6	8·80	9,635	6,840	14,000	10,800	1,800	1,350
7	10·26	7,080	5,026	14,400	10,800	1,800	1,350
8	11·73	5,420	3,848	14,400	10,800	1,800	1,350
9	13·20	4,282	3,040	14,400	10,800	1,800	1,350
10	14·66	3,468	2,462	14,400	10,800	1,800	1,350
13·5	19·9	1,900	1,350	14,400	10,800	1,800	1,350

ANIMAL STRENGTH.

We come now to consider the subject of animal strength as a first mover of machinery.

There is a certain load which an animal can just bear, but

* The force of traction on a canal varies as the square of the velocity; but the mechanical power necessary to move the boat is usually reckoned to increase as the cube of the velocity. On a rail-road or turnpike, the force of traction is constant; but the mechanical power necessary to move the carriage, increases as the velocity.

cannot move with it, and there is a certain velocity with which an animal can move but cannot carry any load. In these two circumstances it is clear, that the exertion of the animal can be of no avail as a mover of machinery. These are, as it were, the extremes of the animal's exertion, where its effect is nothing; but between these two extremes, there must be weights and velocities with which the animal can move, and be more or less efficient. It is not difficult to see one general law which will hold, and that is, that the greater the velocity is with which the animal moves, the less will be the load which it can carry; but it is very difficult to ascertain the true measure of the variation of load and speed.

If one man travel at the rate of three miles an hour, and carry a load of 56 lbs., and another move at the rate of 4 miles an hour and carry a load of 42 lbs.

The speed of the first is 3, and the load 56, the useful effect may therefore be estimated as the momentum $= 168$. The other carries only 42 lbs., but travels at the rate of 4 miles an hour; therefore, in the same way, his useful effect will be $4 \times 42 = 168$, the same as before—therefore the effects of these two men are the same. It will not be difficult to show, that in the same time they perform the same quantity of work. For the first will in 6 hours carry 56 lbs. $3 \times 6 = 18$ miles, as he travels at the rate of 3 miles an hour; and if he be supposed to carry a different load, but of the same weight every mile, he will in the 6 hours have carried altogether $18 \times 56 = 1008$ lbs.; but the other carries in the same way, 4 times 42 lbs. every hour, that is 168 lbs. in one hour—therefore in 6 hours he will have carried $168 \times 6 = 1008$ lbs., the same as the other.

It will be now seen what is meant by the phrase useful effect, and from what has been observed above, we will be led to conclude, that when the load is the greatest which the animal can possibly bear, the useful effect is nothing, because the animal cannot move; and when the animal moves with

its greatest possible speed, the useful effect will also be nothing, for then the animal can carry no load; and it becomes a very useful problem to determine where between these two limits, the load and speed are so related that the useful effect of the animal will be the greatest. The following little table will be found very useful in such enquiries—where the column V shows the velocities, L the load, and U the useful effect.

V.	L.	U.	V.	L.	U.
0	225	0	8	49	392
1	196	196	9	36	324
2	169	338	10	25	250
3	144	432	11	16	176
4	121	484	12	9	108
5	100	500	13	4	52
6	81	486	14	1	14
7	64	448	15	0	0

On looking over this table it will be seen, that the greatest useful effect is produced by a velocity of 5 with a load 100. Now, it is assumed in this table, that the greatest velocity with which the animal can move is 15, (without a load;) the velocity then that is best is $\frac{5}{15}$, or $\frac{1}{3}$ of the greatest velocity; and the load is $\frac{100}{225}$, or $\frac{4}{9}$ of the greatest weight which the animal can bear.

The numbers in the above table do not refer to any particular measure, but may be applied to any measure, by the following considerations:—Take the space through which a man, horse, or any animal can travel in any given time without a load, and divide it by 15, the quotient will be the value of 1 in the column V; and in like manner the greatest load that can be borne without moving, divided by 225, will give a quotient whose value is 1 in the column L. Thus, if the greatest speed at which a man could travel or run, with-

out a load, be 6 miles per hour, then we have $\frac{4}{15}$ or $\frac{2}{5}$ of an hour, as the value of one or 1 in the column V; and, in like manner, if the greatest load which he can bear, without moving, be $2\frac{1}{2}$ cwt., then this reduced to lbs. is 280 lbs., hence,

$$\frac{280}{225} = \frac{56}{45} = 1\frac{11}{45} \text{ lbs.}$$

as the value of 1 in the line L; wherefore, by the table, the greatest effect of the man will be when his velocity of motion is $\frac{2}{5} \times 5\ \frac{10}{3} = 2$ miles an hour in speed, and the load which he bears is $1\frac{11}{45} \times 100$ lbs. $= 1{\cdot}244 \times 100 = 124$ lbs. = the load.

The French employ a measure of animal action which they denominate a Dynamical unit, which is a cubic metre of water raised to the height of a metre. Hence the number of Killogrammes that are raised one Killometre, are dynamical units.

There are so many causes operating to produce variations in animated beings even of the same kind, that it is difficult if not impossible to form a correct estimate of the amount of any one particular class, or the comparative strength of different classes,—hence we find great difference in the results of different experimenters.

So many very different results have been given by different experimenters on the strength of men, that it is next to impossible to fix on any one of these to be employed in our estimate. Gregory has estimated the average force of a man at rest to be 70 lbs., and his utmost walking velocity, when unloaded, to be 6 feet per second; and that a man will produce the greatest mechanical effect in drawing, when the weight was $31\frac{1}{4}$ lbs., with a velocity of 2 feet per second. But this is not the most advantageous way of applying the strength of men, although it has been found to be the best way of employing the strength of horses. Robertson Buchanan states, that the mechanical effects of men in working a pump, in turning a winch, in ringing a bell, and

rowing a boat, are as the numbers 100, 167, 227, and 248. According to Hatchette, of a man working at the cord of a pully to raise the ram of a pile engine = 50 dynamical units. A man drawing water from a well by means of a cord 71; a man working at a capstan 116. The dynamical unit being, as stated before, equivalent in English measure to 2208 lbs., or 4 hogsheads of water raised to the height of 3·281 feet in a minute; these things being considered, the above results will give the average strength of men per day.

We meet with similar difficulties in estimating the strength of horses. According to Desaguliers and Smeaton, 1 horse equal to 5 men. According to Bossut 1 horse equal to 7 men. Schulze makes it 14 men; and Bossut states, that 1 ass is equivalent to 2 men. It is also stated by Amontons, that 2 horses yoked in a plough exert a power of 150 lbs.

FRICTION.

We have considered the effects of the first movers of machinery, and we must now direct our attention to the subject of Friction, which, as we have frequently noticed, tends to diminish these effects. On this subject it is not our intention to dwell long, as all the researches that have been hitherto made in this branch of mechanical science, are not of such a nature as to furnish means of deducing satisfactory laws. The resistance arising from one surface rubbing against another is denominated friction; and it is the only force in nature which is perfectly inert—its tendency always being to destroy motion. Friction may thus be viewed as an obstruction to the power of man in the construction of machinery; but, like all the other forces in nature, it may, when properly understood, be turned to his advantage,—for

friction is the chief cause of the stability of buildings or machinery, and without it animals could not exert their strength.

The friction of planed woods and polished metals, without grease, on one another is about one fourth of the pressure.

The friction does not increase on the increase of the rubbing surfaces.

The friction of metals is nearly constant.

The friction of woods seems to increase after they are sometime in action.

The friction of a cylinder rolling down a plane, is inversely as the diameter of the cylinder.

The friction of wheels is as the diameter of the axle directly, and as the diameter of the wheel inversely. The following hints may be of use for the purpose of diminishing friction.

The gudgeons of pivots and wheels should be made of polished iron; and the bushes or collars in which they move should be made of polished brass. In small and delicate machines, the pivots or knife edges should rest on garnet. Oily substances diminish friction—swine's grease and tallow should be used for wood, but oil for metal. Black lead powder has been used with effect for wooden gudgeons. The ropes of pulleys should be rubbed with tallow.

As to the friction of the mechanic powers. The simple lever has no such resistance, unless the place of the fulcrum be moved during the operation. In the wheel and axle the friction on the axis is nearly as the weight, the diameter of the axis, and the angular velocity—it is however very small. When the sheaves rub against the blocks the friction of the pully is very great. In most, if not in all screws, the friction of the screw is equal to the pressure—the square threaded screw is the best.

In the inclined plane, the friction of a rolling body is far less than that of a sliding one.

To estimate the amount of the friction of a carriage upon a railway, we have,

$$P - \frac{P \times T}{t} = \text{friction},$$

in which rule P signifies the power that will keep the waggon on the plane, independent of friction; T the time of descent without friction,—both of which are to be determined by the laws of the inclined plane given before: and t is the time of actual descent of the waggon or carriage.

There is a loaded carriage on a rail-road 120 feet in length, having an inclination of one foot to the hundred. The carriage, together with its load, weighs 4500 lbs. Now, the height of the plane may be found by the principles of geometry, from the proportion of similar triangles.

100 : 120 :: 1 : 1·2 = the height of the plane; and by the laws of falling bodies, and the properties of the inclined plane.

$\left(\frac{1 \cdot 2}{16}\right)^{\frac{1}{2}} \times 120 = \cdot 2731 \times 120 = 32 \cdot 772 =$ the time in seconds in which the carriage would descend down the plane without friction—and by the properties of the inclined plane, 100 : 1 :: 4500 : 45 = the force that sustains the carriage, without friction, from rolling down the plane: wherefore, by the rule,

$45 - \frac{45 \times 32 \cdot 772}{60} = 20 \cdot 421 =$ the friction in pounds, which retards the carriage in rolling down the railway.

THE ELEMENTS OF MACHINERY.

Having now considered the general laws of motion, the elementary machines and the principal sources of power, we will next direct the attention of the reader to the best contrivances that have hitherto been made for converting one species of motion into another species.

The subject of this chapter may be arranged as follows:

Contrivances in which the acting part have no permanent connexion—comprehending cylindrical wheels and pinions, bevelled wheels, crown wheels—rackwork, ratchet wheels—wheels driven by belts or bands—rag wheels and chains—axles, gudgeons, and chains—contrivances for locking and unlocking machinery—endless screw, lever of Lagaroust, springs, &c. Contrivances in which the communicating parts have a permanent connection—comprehending single, double, triple, and variable cranks—universal joint, sun and planet wheel—ball and socket, arched head and chain—parallel motion, &c. &c.

When a wheel with a rough surface is put in motion round its axis by means of machinery, that motion may be communicated to another wheel on a parallel axis, having also a rough surface; and the two wheels placed so that their cylindrical surface shall be in contact,—then, by reason of the friction, the one wheel will cause the other to revolve. We have seen buff belts fastened to the rims of wheels for this purpose with good effect. It is easy to see that the velocity of the first wheel will be to the velocity of the second, inversely, as the diameter of the first is to the diameter of the second; so that if the second axle is to move with

three times the velocity of the first, the diameter of the second wheel must be one third of the first.

When the motion of the first axle is to be communicated to the second, which is not parallel but inclined at some angle to it, then the wheels are not cylindrical, as in the above case, but formed like cones, and are called bevelled wheels or geer. The wheels are in this case the frustum of cones, whose bases are the broad surface or faces of the wheels, and whose vertices meet in the point where their axles would meet if produced. This will be easily understood from an examination of fig. 98. The same proportion of velocities must be observed here as above.

Motion is frequently communicated from one axle to another at right angles to it, by forming the teeth of the wheel at right angles to the plane of motion, and causing them to act on a pinion, as in fig. 99. Such a contrivance is called a crown wheel, and may be seen exemplified in the common watch.

When the teeth on the rim of a wheel are made to act on teeth cut on the face of a straight bar, so that the circular motion of the wheel may cause the rod to move backwards or forwards, the contrivance is called rackwork.—(Fig. 100.)

The teeth of a wheel are sometimes formed like saw teeth, and adjusted in such a way, that by means of a clack acting on them, the wheel can only move in one direction. Such a contrivance is called a ratchet wheel, or sometimes, as in clocks, a detant. This will be easily understood from fig. 101.

Motion is often given from one wheel to another by means of a belt or rope. If the belt or rope does not cross, the wheels will both move the same, but if the belt crosses in its passage from the one to the other, then they will move in contrary directions. When belts or straps are used, the cylinders on which they act when broad, are called drums; and they are usually made of the form of double cones, being thickest in the middle, from this circumstance, that the

belt has always a tendency to move to the thickest part of the drum. When the belt becomes stretched by use, and consequently slack, it becomes ineffective, because its friction is diminished—a defect which is remedied sometimes by shortening the belt, sometimes by chalking it, and sometimes by increasing the diameter of the wheels round which it passes. This last remedy is frequently effected by forming the wheels of different diameters in different parts.

When there is great strain, chains are often used instead of ropes, and the links of the chain are made to act on prominences on the drums, or sometimes by sharp studs on the chain. Such contrivances are called rag wheels with chains.

The use of axles and gudgeons has been before explained. It is adviseable, that the gudgeons, or cylindrical ends of axles or shafts should not be very long. Axles are generally fixed firmly to the wheels with which they are connected, but an improvement has been made by Mr Brunell which seems worthy of attention. He formed the axle of a conical shape, as also the hole in the centre of the wheel; and by driving the wheel to a thicker part of the shaft he thus increased the friction at pleasure: and he tempered it so, that when there was no extra resistance in the working of the machine, the wheel kept firmly on the axle—but when any unusual obstacle was opposed to the working of the machine, the friction of the wheel and axle was overcome, and the wheel and axle moved no longer together. This principle might be usefully employed in the prevention of accidents in large factories. It may be observed, that when the ends of axles are of a conical shape, they are called pivots, and the sockets in which gudgeons act are called bushes.

In connected machinery, it becomes often necessary that some part of the train should be stopped while the rest goes on; or a department, previously at rest, brought into action, and that without checking the rest of the machinery,—hence we have contrivances for throwing in and out of geer,

as it is called, or the locking and unlocking of machinery. This may be done in various ways, one of which is the following:—Upon one axle there is fixed a conical shoulder, which is so contrived, that it may be slid backward and forward but cannot revolve. This conical shoulder is moved by means of a lever into a hole in the unlocked wheel, which is made so as exactly to fit it; and the friction arising from the pressure of the shoulder on the hole of the wheel will cause them both to revolve together. The machinery which this will drive will then be brought into train, and may be taken off by moving back the shoulder.

When it is required to produce a circular motion, which shall be at right angles to a given circular motion, the endless screw may be employed,—its use and construction will be easily understood from the figure.—(See fig. 102.)

When it is required to produce motion in a straight line, from motion in a circle, the lever of Lagaroust may be employed. (See fig. 103) MN is a fixed beam, in which is fastened the pin C, which is the fulcrum round which the lever AB turns, which carries with it two other levers DE D′E′, whose fulcrums are DD′. The rack rods have teeth of a saw shape, so that when the arm A of the lever descends, the hook E slides over the tooth, while the other hook E′ catches the tooth and raises the rack, and so on with the other side, when the lever AB comes into the opposite position.

Springs are frequently used in machinery. Spiral springs often connect the ends of chains, in order to tighten them when they happen to get slack.

We come now to speak of the contrivances, in which the communicating parts are permanently connected.

The crank whose form is well known, though not the most efficacious, is nevertheless one of the simplest and most lasting of all modes of communicating motion. The power acts on the crank on the same principle as the winch, and it

it is generally used for converting a rectilinear into a rotatory motion. The knee of a crank describes the circumference of a circle, and if the power which moved it acted always as a tangent to that circle, there would be no loss of motion in the crank; but the connecting rod acts always nearly parallel to itself, in consequence of which there is a loss of power. See fig. 104, where *c* is the centre of motion. When the crank and connecting rod are in one right line, all the pressure of the rod will be exerted on the centre *c*, and the crank will have no tendency to turn round; but when the crank is turned a little to the one side, the rod will turn on the principle of the lever as at *n*, where the crank is a lever whose length is *nm*; when the crank is at the quarter circle *q*, then the leverage and power of the connecting rod will be greatest; and as the crank moves round, it will get less and less until it arrives at the bottom, where, as in the first position, the power will be nothing; but as it is exerted in a direction contrary to that of the crank's motion, will be a positive drawback to the crank's power. When there are two connecting rods and two cranks, as in a steam boat's machinery, it is common to place the one crank in a position on the shaft at right angles to one another, so that when the one crank is in the least effective position, the other may be in the greatest, and thus the power of the one assists the resistance of the other. But it will be adviseable to place them at an angle rather greater than a right angle, where the sum of the lengths of the levers, formed by the crank, in all positions will be greater. In this arrangement, when the one crank is in the least effective position, the other will have sufficient power to turn it, although it is not at its most powerful point, as it will be a little past the quarter circle; but this loss will be more than compensated, as the sum of their powers, in all other points, will be greater than if they were placed at right angles to each other. Sometimes three cranks are employed, and when this is the case, they are commonly placed at an angle of 120 degrees to each other.

When it is wished to communicate motion from one axis to another at any given angle, Hook's universal joint may be employed. The two axles AB fig. 105, between which the motion is to be communicated, have strong semicircular hoops attached to their ends; the diameters of which hoops, CD and EF, are fixed in the form of a cross—their extremities moving freely in bushes placed in the extremities of the semicircles. Thus, while the central cross remains unmoved, the shaft A and its semicircular end, may revolve round CD as an axis; and the shaft B and its semicircular end, may, in like manner, revolve round EF as an axis. If the shaft A be made to revolve without changing its direction, the points CD will move in a circle whose centre is at the middle of the cross. The motion thus given to the cross, will cause the points EF to revolve in another circle round the same centre—and hence the shaft B is made to revolve. This is what is called the single joint, and will not apply when the inclination of the two shafts is less than 140 degrees; but the double joint will apply to all angles. The double joint is just two of the above joints interposed between the shafts. The driving shaft moving a joint which moves another joint, which again moves the driven shaft. In cotton mills, where the shafts should never be very long, this joint is frequently used.

The sun and planet wheel is an ingenious contrivance of Mr Watt, for converting a rectilinear into a circular motion, which he applied to the steam engine. On the end of the connecting rod, there was fastened a teethed wheel firmly fixed, so that it could not move round its own axis, and its teeth were made to act on the teeth of another wheel of the same size, fixed on the shaft of the fly; and thus by the motion of the connecting rod, motion was given to the fly wheel.

The ball and socket may here be noticed, though it is not frequently used in machinery. A lever is fixed on a ball,

which rests in a hollow hemisphere, to which it is adapte and by this means the fulcrum is formed into a kind of uni versal joint, whereby the ends of the lever may be pointed i almost every direction. Its most common applications ar in small telescopes mounted on stands, and in the machine used for taking profiles, called the phisiognatrace.

Having made these introductory statements, we will now proceed to consider more at large, the various means of transferring or directing motion.

The most simple contrivance for transferring a direct circular motion into another of the same kind, in the same plane, is by means of two wheels, round which a belt, rope, or chain, passes without crossing. The same thing may be effected by three teethed wheels acting on one another, where, if the first wheel turn from right to left, the middle wheel will turn from left to right; and the last wheel from right to left the same as the first. The same thing may be produced by cutting teeth on the rim of one wheel on the interior side of the rim facing the centre; and causing them to act on similar teeth cut on the outside of the rim of another wheel—then these wheels will turn the same way.

A direct circular motion may be converted into a circular motion in the contrary direction, by simply causing one toothed wheel to act on another, when the teeth of both are on the exterior of the rim. The same thing may be effected by causing a belt which passes round the wheels to cross between them.

When the circular motion to be communicated is not in the same plane, but inclined, bevelled wheels may be employed—the universal joint, or the endless screw; all of which have been mentioned above. The last of these, the endless screw is commonly employed to communicate motion from one plane to another at right angles to it; but it may

be used at any angle by simply bevelling the wheel. It must be remembered, however, in all cases, that the plane of the teeth of the wheel must be parallel to the axis of the screw.

Contrivances for transferring alternating vibrating motion, have been made by Camus and others, which are sufficiently simple. For the purpose of moving sieves, Camus placed a plank moving round two gudgeons at the ends, to the end of the plank there was attached a bent rod, which passed through a bush, supported from the end of the table. To the end of the rod there was fixed a pendulum, the vibratory motion of which was communicated to the plank, and motion in this way given to the sieves. Another contrivance for the same purpose is exemplified in the old form of the turning lathe, where a spring pole was fastened at the roof, to the end of which there was fastened a cord, which passed round a wheel below the pole, and the cord below this was fastened to a traddle, and by the motion of the traddle, and the elasticity of the spring, an alternating circular motion was given to the wheel.

We now come to the consideration of the methods of changing one kind of motion into another of a different kind.

The conversion of a continuous rectilinear motion into a continuous circular motion, may be effected in various ways. If a weight hang at the end of a cord, which is coiled round a wheel, then the weight descending by the force of gravity in a right line, will uncoil the string, and give to the wheel a circular motion round its axis. If a straight bar, having teeth upon it, be moved in a right line, and be so adjusted that its teeth act in the teeth of a wheel, then will the rectilinear motion of the bar or rack give a revolving motion to the wheel, or the revolving motion of the wheel may give a rectilinear motion to the bar. This is illustrated in the

piston rods of the common air pump. The same thing may also be done by causing a belt or chain, which moves in a right line, to act on the rim of a wheel like a tangent to a circle. The same thing is also done very elegantly, by causing a male screw to turn round, on which there is placed a female screw or nut, which is prevented from turning round, as it moves in a groove; and thus the turning of the screw causes the nut to move parallel to the axis of the screw.

There are various contrivances for converting a rectilinear motion into a circular alternating one. One of these is the following:—There is a lever having a long and a short end, the long end acts in the teeth of a rack, which moves up or down; one tooth of the rack catches the lever and raises it, but when it goes a certain height, the catch of the tooth is lost, and the heavy end of the lever falls, till it is caught by another tooth, and again raised, and so on; the right lined motion of the rack produces an alternating circular motion of the lever round its fulcrum. The same thing may be done by using a teethed wheel instead of a lever, and a chain passing over two wheels instead of a rack, as in fig. 106.

The lever of Lagaroust may be employed to produce a like effect, when its action, as it was before described, is inverted, that is, when the rack is made the first mover.

The only contrivance which we think capable of producing a rectilinear alternating motion, from a rectilinear continuous motion, is the following:—A chain AB (fig. 107) has a rectilinear motion, in the direction AB, by means of rag wheels; and is furnished with wipers *m*, *n*, *o*, which, when they are moved, will obviously raise the arm CD through a certain height. As soon as the wiper quits it, it will descend by its own gravity; and will then be raised by the next wiper *m*, thus acquiring an alternating rectilinear motion.

Of contrivances for converting a continuous circular motion into an alternating rectilinear one, we have many in

which great ingenuity is exhibited. We will confine our attention to the most simple of these.

One method of doing this, is by causing a beam or stamper to rest on a wheel which is irregularly formed at the circumference, as in the heart wheel, or wheel with wipers, (see fig. 108.) Thus the stamper is raised to a certain height, and again descends by its own weight. The very same thing may be better done by a rack and pinion, but the pinion must want teeth on that part where the rack is meant to descend. This is sufficiently simple, and requires no more minute description. A contrivance for the same purpose, but more complicated and extensively useful, is seen in fig. 109, where AB is a wheel turned either by a winch or belt; this wheel gives a rectilinear motion to the beam CD, by means of a lever which connects the end of the beam with the circumference of the wheel. The beam moves between two guides, and it is clear from an inspection of the figure, that the greatest range of the beam will be the diameter of the circle described, by the end of the lever attached to the wheel. This contrivance is employed horizontally in grinding flat surfaces, and also in silk mills.

By means of a pinion, one half of whose circumference is stripped of teeth, and a double rack, the same thing may be effected. From a slight inspection of the figure 110, this will be easily understood. Suppose that the pinion turns from left to right, its teeth will catch on the teeth on the left side of the rack, and raise the rack; but from the want of teeth on the pinion, the other side will be free—but continuing to turn, its teeth will catch the rack on the left side and depress it, the left side being free. There may be various modifications of this contrivance, which will readily suggest themselves to the attentive reader. One beautiful modification of this is shown in fig. 111, where the pinion wants none of its teeth. The pinion does not fill the rack, but, as may be seen in the figure, will, by its turning from

[illegible] shown, cause the rack to [illegible]: the pinion acting still [illegible] move to the one side, and thus [illegible] into contact, and so at the [illegible] rack's motion sidewise, it is [illegible]

[illegible] very properly termed [illegible] on this very simple and [illegible] circular piece of metal there [illegible] diverging from the centre; [illegible] was placed another plate which [illegible] shown in the figure 112. [illegible] placed through the radii [illegible] through the spiral of the other, [illegible] spiral plate, would cause the [illegible] the centre of the first [illegible] of this kind are placed together, [illegible] with the rod which passes through [illegible] through a radius of the other: [illegible] of these rods, which [illegible] according as the radii and spiral [illegible] each other.

[illegible] was the contrivance of Zurida, for the purpose of [illegible] for cards. He made a wheel [illegible] of a crown wheel, but with saw teeth: against these teeth a lever was pressed by means of a spring, and the turning of the wheel thus caused a backward and forward motion of the lever. To the same author we are indebted for another very beautiful contrivance belonging to this class, on a cylinder, (see fig. 113.;) he cut two screws, the one left and the other right handed, which run into each other at the ends, but crossed every other where, as may be seen in the figure. Now a pin was placed in the groove of the screw, which moved in a guide running parallel with the axis of the cylinder. When the cylinder was turned round

the pin, would be moved forward by reason of the thread of the screw; when the pin came to the end, it would, by the nature of the thread, come into the contrary thread, and thus move in the opposite direction.

In fig. 114, there is shown another contrivance, where a pinion acts on a rack, which is teethed on its edges and ends; when the pinion, by turning round, causes the rack to move to the left, then there is a spring which presses it to the pinion, so that the teeth at the end continue to act, and the axis of the pinion is capable of sliding up or down, to accommodate itself to the upper or under surface of the rack.

If one wheel be made to move within another of twice its diameter, the teeth on the exterior of the rim of the small wheel acting on the teeth on the interior of the rim of the large one; then the small wheel being moved, any part of its circumference will describe a straight line. This has been employed as a parallel motion to the steam engine. (fig. 115.)

We have several contrivances for converting a continuous circular motion into an alternating circular one.

This is effected in raising a forge hammer, moveable about a fulcrum, by means of wipers, on the circumference of a wheel; when the hammer is raised by the wiper depressing the other end of the lever, and when the wiper lets go, the hammer falls by reason of its own weight. The same thing may be done by causing the wipers of a wheel to strike the bob of a pendulum, as in fig. 116. A simple contrivance is shown in fig. 117, where there is a crown wheel, with teeth only on one half of its circumference. At right angles to the shaft of this wheel, there is another shaft, having two wheels, on which the teeth of the crown wheel act. Now, when the crown wheel turns round, it is clear that it can only act on one of the other wheels at a time; and by one revolution, will cause the one wheel to move one way, and the other the contrary; and thus an alternating circular motion is effected from a continued one. For similar pur-

poses, we may employ the crank—a modification of the lever of Lagaroust—the sun and planet wheel, and various escapements for clock work.

The mechanical contrivances for converting rectilinear alternating into circular alternating motion, ought, from their extensive application, to be carefully studied by the engineer.

A simple contrivance of this kind is represented in fig. 118. It consists of a lever with a semicircle on its under face: to the joining of this semicircle with a lever, a rope is fastened, which passes round two sheaves; the lever having an alternating motion, the rope will have a motion backwards and forwards between the two sheaves, which will easily be understood from the figure.

The common drill and bow is another example of the same species of contrivance, where the backwards and forwards motion of the bow gives an alternating circular motion to the drill.

The piston rod of a steam engine may be made to move up and down in a right line in various ways. The rod may be made to terminate in a rack, the teeth of which act in the teeth of an arched head of the long lever, called the working beam: but the most efficacious of all contrivances of this kind, is that of Watt, commonly called the parallel motion. This contrivance is founded on geometrical principles, which it would be inconsistent with the plan of this work to consider; we shall therefore simply describe the contrivance of this illustrious mechanic.—(See fig. 119.)

The working beam has an alternating circular motion round its centre A, and it is clear that the points B and G will have a circular motion round the common centre A. Let the point B be exactly in the middle, between the centre and end of the beam. Let there be a bar or rod CD, of the same length as AB, capable of moving round the centre C, by means of a pivot. The other end of this rod is attached,

by means of a pivot, to the rod DB. Now, by the alternate rising and falling of the beam, the points B and D will move in circular arches, but the middle point P, of the connecting rod BD, will move upwards and downwards in a vertical straight line, or at least so very nearly so, as the difference cannot be perceived. Now, to this point P, there is attached the end of the pump rod, which will, of course, follow the direction of the impelling point, and move in a straight line. For the purpose of communicating a similar motion to the other piston rod, conceive another rod GP′ is introduced, of the same length as BD, and its extremities moving likewise on pivots. The piston rod of the cylinder is attached to the point P′, and this point moves quite in the same way as the point P. The only difference in the motion of these two points will be, that the point P′ will move twice as fast as the point P, or will, in the same time, move twice as far.

OF MACHINERY IN GENERAL, AND THE MEANS OF ITS REGULATION.

A machine, howsoever complicated it may be, is nothing else than an organ or instrument placed between the workmen, or source of force or power, whatever it may be, and the work to be done. Machines are used chiefly for three reasons.—1. To accommodate the direction of the moving force, to that of the resistance which is to be overcome. 2. To render a power, which has a fixed and certain velocity, effective in performing work with a different velocity. 3. To make a moving power, with a certain intensity, capable of balancing or overcoming a resistance of a greater intensity.

These objects may be accomplished in different ways,

either by using machines which have a motion round some fixed point, as the three first mechanic powers; or by those which furnish, to the resistance to be moved, a solid path along which it may be impelled, as is the case in the last three mechanic powers,—hence some authors have reduced the simple machines to two—the lever and inclined plane. Simplicity in the construction of machines cannot be too warmly recommended to the young engineer; for complexity increases the friction and expense, and endangers the chance of success from the derangement of the parts. In consequence of friction, it is well known, that no machine can overcome a resistance without an expense of the power of the first mover, and as the more complicated the machine is the greater will the friction be; so also will the machine be less powerful. If two machines be constructed, the one simple and the other complex, and be such, that the velocity of the impelled point is to the velocity of the working point in the same proportion in both; then will the simple machine be the most powerful.

The methods of communicating motion from one point to another are infinitely diversified; and we, in the last chapter, gave an account of the best of these which have hitherto been invented. We confine ourselves in the meantime to a few general remarks on the construction of machinery.

When heavy stampers are to be raised in order to drop on matter to be pounded, the wipers by which they are raised, should be of such a form, that the stampers may be raised by a uniform pressure, or with a motion as nearly as possible uniform. If this is not the case, and the wiper is merely a pin sticking out of the axis, the stamper will be forced into motion at once, which will occasion violent jolts in the machine, together with great strains on its moving parts, and points of support. But if gradually lifted, no inequality will be felt at the impelled point of the machine. The judicious engineer will therefore avoid, as much as possible, all sudden

changes of motion, especially in any ponderous part of a machine.

When several stampers, pistons, or other reciprocal movers are to be raised and depressed, common sense teaches us to distribute their times of action in a uniform manner, so that the machine may be always equally loaded with work. When this is done, and the observations in the foregoing paragraph attended to, the machine may be made to move almost as smoothly as if there were no reciprocations in it. Nothing shows the ingenuity, or skill of the contriver, more than the simple yet effectual contrivances, for obviating those difficulties which are unavoidable, from the nature of the work to be done by the machine, or of the power applied. There is also much ingenuity required in the management of the moving power, when it is such as does not answer the kind of motion required; for instance, in employing a power which necessarily reciprocates to produce a motion which shall be uniform, as in the employment of a steam engine to drive a cotton mill. The necessity of reciprocation of the first mover, causes a waste of much power. The impelling power is wasted first in imparting, and then in destroying a vast quantity of motion in the working beam. The engineer will see the necessity of erecting the mover in a separate building from the machinery moved, which prevents the great shaking and speedy destruction of the buildings.

The gudgeons of a water wheel should never rest on the building, but should be placed on a separate erection; and if this is not practicable, blocks of oak should be placed below them, which tend to soften all tremors, like the springs of a carriage.

It will often conduce to the equality of motion of machinery, to make the resistance unequal, to accommodate the inequalities of the moving power. There are some

beautiful specimens of this kind in the mechanism of the human body.

It is always desirable, that the motion of a machine should be regular, when this can be effected; and we now proceed to state the various methods that have heretofore been employed for producing regularity in the motion of the machine, both as regards the reception and distribution of power.

Even supposing that the first mover is perfectly constant and equable in its action, the machine may not be regular in its motion, from the irregularity of the resistance to be overcome. But still, if both the power and the resistance were perfectly regular, the machine would not be perfectly uniform in its motion; for there are particular positions in which the moving parts of a machine are more efficacious than in others, as in the crank for instance: hence the energy of the first mover will be unequally transmitted, and irregularity in the motion of the machine will consequently follow. The motion of some machines bears a constant tendency to accelerate, others to retard; and others alternately to accelerate and retard; and perhaps in no case whatever can the motion of a machine be said to be perfectly uniform. But of this we will speak more at large when we come to treat of the maximum effect of machines.

We intend to confine our attention chiefly to the regulators of machinery employed in the steam engine, making occasional remarks on others as we go along.

For the purposes of regulating the moving power, the conical pendulum or governor is commonly employed. The nature of this beautiful contrivance has been described under central forces, and alluded to in our remarks on the steam engine. The ring on the shaft acts upon a lever of the first kind, whose other end opens or shuts a valve, which is fixed in the pipe that admits the steam from the boiler to the cylinder; and according to the degree of opening or

shutting of this valve, and consequently the divergence or convergence of the balls, or the velocity of the shaft, will be the quantity of steam admitted to the cylinder. The governor is frequently applied to the water wheel, and acts in a similar way by a board or valve in the shuttle, which delivers the water to the wheel. So likewise in the windmill, it is employed to furl or unfurl more or less sail.

Sometimes the governor is found inadequate to the regulation of the machine, and another contrivance of great power and simplicity is introduced. The machine is made to work a pump, which tends continually to fill a cistern with water. From this cistern there proceeds an eduction pipe, leading to the reservoir, from which the water is drawn by the pump. This simple contrivance is so regulated, that when the machine goes with its proper velocity, the pump throws just as much water into the cistern as the ejection pipe draws from it; consequently, the water in the cistern remains at the same level. But if the machine goes too fast, then the pump will throw in more water than is let out by the ejection pipe, wherefore the level of the water will rise in the cistern. If the machine goes too slow, the level of the water will in like manner fall. Now, on the surface of the water in the cistern, there is a float which rises or falls with the surface of the water; and is thus made to answer the same purpose as the ring of the governor. It may be observed, that the delicacy of this kind of regulator will depend, in a great measure, upon the smallness of the surface of the water which supports the float; for then a small difference between the supply and discharge, will cause a greater difference in the elevation or depression of the surface, than if the surface were large.

To procure a constant supply of steam in the steam engine, it is necessary that the water in the boiler be always at the same level. To effect this purpose, there is a lever fixed on a support, on the top of the boiler, to one end of which

lever there is attached a slender rod, which descends into the boiler, and is terminated by a float, which rests on the surface of the water in the boiler. To the other end of the lever, there is attached another rod, to the end of which is affixed a valve, opening and shutting the orifice of a pipe which leads into the boiler. The top of the pipe, where the valve is placed, opens into a cistern of water, which is supplied by a pump driven by the engine itself. When the water in the boiler falls below its common level, in consequence of the formation of steam, the float falls with it, and consequently depresses that side of the lever to which the float rod is attached; the other arm rises and opens the valve at the top of the pipe, which leads from the cistern into the boiler, and thus admits water until the float rises to the proper height, and then the valve is closed. All this will be instantly understood from an inspection of the fig. 119. In this beautiful contrivance, the water is not supplied to the boiler in jolts, but the float and valve continuing in a state of constant and quick vibration, the supply is rendered quite constant.

There is a very ingenious contrivance called the Tachometer, from its use as a measure of small variations in velocity, which is often employed in the steam engine and other machinery. The simplicity of this contrivance will render its action easily understood. If a cup with any fluid, as mercury, be placed on a spindle, so that the brim of the cup shall revolve horizontally round its centre, then the mercury in the cup will assume a concave form, that is, the mercury will rise on the sides of the cup, and be depressed in the middle; and the more rapid the motion of the cup is, the more will the surface of the mercury differ from a plane. Now, if the mouth of this cup be closed, and a tube inserted in it, terminated in the cup by a ball-shaped end, and half filled with some coloured fluid, as spirits of wine and dragon's blood; then it is clear, that the more the surface of

the mercury is depressed, the more the fluid in the tube will fall, and *vice versa*: consequently, the rapidity or slowness of the motion of the cup, will be indicated by the height of the coloured fluid in the tube; and thus it becomes a measure of small variations in velocity.

In the steam engine, we also find an apparatus for regulating the strength of the fire of the boiler, which apparatus is called the self-acting damper. There is a tube inserted into the boiler, reaching nearly to the bottom, which tube is open at both ends. Now, from the principles of Pneumatics, it is plain, that the greater the pressure of the steam in the boiler is, the water will be pressed to the greater height in this tube. The water in the tube supports a weight, to which there is attached a chain going over two wheels; and to the other end of the chain is attached a plate, which slides over the mouth of the flue which leads into the fire. These things are so formed, that the rising of the weight in the tube will cause more or less of the flue to be covered by the plate; and thus increase or diminish the quantity of air which feeds the fire. Now, if there is too much steam produced, there will be a greater pressure on the surface of the water in the boiler, and it will be forced up the tube—the weight in the tube will be raised, and consequently the plate at the other end of the chain will fall, and cover more of the mouth of the flue, and thus diminish the quantity of air which feeds the fire; and there will consequently be generated in the boiler a less quantity of steam.

We come now to speak of the nature and use of the fly wheel. A fly in mechanics may be defined to be a heavy wheel or cylinder which moves rapidly upon its axis, and is applied to a machine for the purpose of regulating its motion.

We have already stated that there are many circumstances

which tend to render the motion of a machine irregular—variation in the energy of the first mover, whether it be water, wind, steam, or animal strength—variation in the resistance or work to be done—and variations in the efficacy of the machine itself, arising from the nature of its construction, whereby it is of necessity more effective in one position than in another. We have already seen how many of these irregularities are compensated, and we are now come to speak of the fly, which is the simplest and most effective of them all. The principle on which the fly acts is that of inertia, one of the most important of the first principles of mechanical science. At any one given time, a body must be in one or other of these two states—rest or motion. And let any body be in one or other of these two states, it has no power within itself to change it,—if it be at rest, it has no power to put itself in motion—and if in motion, it has no power in itself either to increase, diminish, or destroy that motion. From a knowledge of this fact, and from what was stated before on the momentum, or moving force of a body, that it is the quantity of matter multiplied by the velocity of the moving body—the nature of the operation of the fly will be easily understood.

As the fly wheel, to do its office effectually, must have a considerable velocity, it is clear that its rim, which has a considerable weight, must also have a considerable momentum, and consequently a considerable power to overcome any tendency either to increase or retard its motion.

To apply these observations to actual cases, let us suppose that a single horse drives a gin. When the gin has been set in motion, the animal cannot exert a uniform strength—there will be occasional increases and relaxations in the velocity of the gin; but suppose a fly wheel to be added, then, whenever the animal slackened its exertions, the machine would have a tendency to move slower, but the momentum which the fly had acquired, would overcome this

tendency to retardation, and continue the motion of the machine at the same rate as before, until the animal had recovered itself so as to exert the same strength as before. So, likewise, if the animal exerted an extraordinary pull, the inertia of the wheel would oppose a resistance which would check the tendency to increase in the velocity of the gin. In this way the fly wheel regulates the motion of the gin, whether the animal takes occasional rests, or makes occasional extraordinary exertions. It is evident that the fly would operate in the same way, if the first mover were steam, water, or wind, and that the other regulators which we have described, are merely assistants to the fly wheel.

Variations in the resistance, or work to be performed, are also compensated by the fly wheel. For instance, in a small thrashing mill without a fly. When the machine is not regularly fed with the corn, there will be an occasional resistance, which will have a sensible effect on the whole train of the machinery, even the water wheel itself; which irregularity may, however, be avoided by the introduction of a fly, as its inertia will procure equality of motion: but it may be observed, that when the machine is large, there will be less necessity for a fly, as the inertia of the machine itself will then effect the same purposes.

It was before stated, that even supposing the first mover and resistance to be perfectly uniform, the machine itself is liable to variations in energy at different positions. It was seen, for instance, that a crank is more effective in one position than another; but the momentum communicated to the fly, when the crank is in the most effective position, will carry the crank past its least effective position. There are many cases, however, where there are irregularities of motion proceeding from the nature of the machinery, which could be compensated better than with a fly. Thus, if a bucket is to be drawn from the bottom of a coal pit, which is 60 fathoms in depth: the weight of the bucket being 14

cwt., and the chain by which it is coiled up round the cylinder weighing 8 lbs. to every fathom,—it is plain, that when the bucket is at the bottom, not only the weight of the bucket, but also the weight of the chain, will require to be overcome in the raising of the bucket. Now the weight of the chain is $60 \times 8 = 480$ lbs., and the amount of the weight of the bucket is 14 cwt. or 1568 lbs.; hence $1568 + 480 = 2048$ lbs.; but the weight of the chain will always be getting less as it is coiled round the cylinder, until the bucket comes to the cylinder, when the chain will be all coiled, and there will remain only the weight of the bucket. Now, the use of a fly may be advantageously dispensed with, if the barrel on which the chain is coiled is formed like a cone; the diameter of the barrel thus increasing with the uniform diminution of the weight.

The effect of the fly wheel in accumulating force, has led some to suppose that there is, positively, a creation of force in the fly; but this is a mistake, for it is only, as it were, a magazine of power, where there is no force but what has been delivered to it. The great use of the fly wheel is thus to deliver out at proper intervals, that force which has been previously communicated to it; and although there is absolutely a small loss of power by the use of the fly, yet this is more than compensated by its utility as a regulator.

THE EFFECT OF MACHINES.

The motion of machines may, as stated before, be reduced to three kinds. That which is gradually accelerated, which generally takes place at the commencement of a machine's action: that which is entirely uniform: that which is alternately accelerated and retarded. The nearer that the mo-

tion of a machine approaches to uniformity, the greater will be the quantity of work done.

In order that the few remarks, which we intend to make on the effect of machines, may be clearly understood, we desire the reader to attend to the following definitions.

The impelled point of any machine, is that point at which the force which moves the machine, may be considered as applied—as the piston of a steam engine, or the float board of a water wheel.

The working point, on the contrary, is that point where the resistance may be supposed to act.

The velocity of the moving power is the same as the velocity of the impelled point,—the velocity of the resistance is the same as the velocity of the working point.

The performance or effect of a machine is measured by the resistance or work performed, (calculated by weight,) multiplied by its velocity which is, in other words, the momentum of the working point. The momentum of impulse, on the other hand, is measured by the energy of the first mover, (also estimated by weight,) multiplied by the velocity of the impelled point.

These definitions being understood, we proceed to a simple statement of principles.

When any power is made to act in opposition to a resistance, by means either of a simple or compound machine; which machine will be in a state of rest, when the velocity of the power is to that of the resistance, as the weight of the resistance is to that of the power. In this state of things the machine can do no work, because it has no motion; but if the power is increased, so as to overcome the resistance, the machine will have an accelerated motion so long as the power exceeds the resistance. If the power should diminish, the machine would accelerate less and less, until its motion became uniform. The same effect would necessarily follow, if, instead of the power diminishing, the resistance increased,

a circumstance which may arise from various causes. From the resistance of the air, which increases with an increase of velocity; and also from friction, which frequently, though not always, increases with the increase of velocity. Hence we find, that machines have commonly a tendency to become uniform in their motion.

We have seen before, while treating of the water wheel, that the velocity of the floats of the undershot wheel, must be less than the velocity of the stream. For, when the float board is at rest, the water will impinge on it with the greatest possible effect; but so soon as the float begins to move, then it leaves the water, as it were, and does not receive the whole impetus of the stream; and if the velocity of the float were equal to that of the stream, it is clear that the water would have no effect upon it at all; and, as was stated before, there is a certain relation between the velocity of the wheel and that of the stream, at which the effect will be a maximum. This is not confined to the water wheel, but is common to all machines, as we have seen illustrated in the steam engine.

We have seen before, that the maximum effect of an animal was, when its velocity was one third of its greatest possible speed, and the load which it bore or the resistance which it overcame, was equal to four-ninths of its greatest possible load.

The following tables constructed from the results of Dr Robison, will be useful to the mechanic.

Table A contains the least proportion between the velocities of the impelled and working points of a machine; or between the levers by which the power and resistance act.

The use of this table is very simple, for suppose we wished to raise 3 cubic feet of water per second, by means of a water wheel, whose radius was 8 feet, (= the length of the lever by which the power acts,) and the power which moves the wheel being 6 cubic feet of water per second.

Employ this rule:

$$\frac{\text{Power,}}{\text{Resistance,}} \times 10 = \text{a number,}$$

which look for in column M, and against it in column N, will be found a number which, when multiplied by the length of lever at which the power acts, will give the length of lever at which the resistance should act.

Wherefore, in the above case,

$\frac{6}{3} \times 10 = 20$, the number corresponding to which is 0·732051, hence $0·732051 \times 8 = 5·856408$ = the radius of the axle at which the resistance or work to be done acts.

This table will be found very useful in the construction of machines; but they are frequently already constructed, and it becomes then necessary for us to regulate the power and resistance in order to produce a maximum effect, without making any alteration in the machine. For this purpose we employ table B, in order to show the use of which we give the following rule and example:

$\frac{\text{Length of lever of resistance,}}{\text{Length of lever of power,}}$ = a number, which, when found in column O, will stand against a number in column P: such, when multiplied by the energy of power, will give the proper energy of resistance. Thus, if a man exerts a constant force of 56 lbs. on the handle of a capstain, whose leverage is 4 feet, and the barrel is one foot in radius, then we have,

$\frac{1}{4} = \frac{1}{4}$ a number, which will be found in column O, corresponding to which will be found, in column P, the number, 1·8885; wherefore, by the rule,

$1·8885 \times 56 = 105·756$ = the resistance which the man,

in these circumstances, can overcome with the greatest advantage, or with the maximum mechanical effect.

TABLE A.

M	N	M	N
1	0·048809	20	0·732051
2	0·095445	21	0·760682
3	0·140175	22	0·788854
4	0·183216	23	0·816590
5	0·224745	24	0·843900
6	0·264911	25	0·870800
7	0·303841	26	0·897300
8	0·341641	27	0·923500
9	0·378405	28	0·949400
10	0·414211	29	0·974800
11	0·449138	30	1·000000
12	0·483240	40	1·236200
13	0·516575	50	1·449500
14	0·549193	60	1·645700
15	0·581139	70	1·828400
16	0·612451	80	2·000000
17	0·643168	90	2·162300
18	0·673320	100	2·316600
19	0·702938		

TABLE B.

O	P	O	P
$\frac{1}{4}$	1·8885	7	0·03731
$\frac{1}{3}$	1·3928	8	0·03125
$\frac{1}{2}$	0·8986	9	0·02669
1	0·4142	10	0·02317
2	0·1830	11	0·02037
3	0·1111	12	0·01809
4	0·0772	13	0·01622
5	0·0580	14	0·01466
6	0·0457	15	0·01333

It is not by any means an easy matter to estimate the relative quantities of work done by different machines. Their effects are generally stated as equivalent to so many horses' power, and the following data are commonly given: One horse's power, at a maximum, is equivalent to the raising of 1000 lbs. 13 feet high in one minute. In cotton factories, 100 spindles, with preparation, are allowed to each horse power for spinning cotton yarn twist, or five times that number of spindles, with preparation, for mule yarn, No. 48; and if it be No. 110, ten times that number of spindles, with preparation—and the power-loom factories 12 beams with subservient machinery.

Thus a steam engine on Watt's principle, having a cylinder of 30 inches diameter, and a stroke of 6 feet, making 21 double strokes per minute, will give, by the usual calculation,

$$\frac{\cdot 7854 \times 30^2 \times 10 \times 6 \times 21 \times 2}{44000} =$$

40 horses' power. Hence such an engine will drive 4000 spindles cotton yarn twist, or 20000 spindles mule twist, No. 48, or 40000 mule twist spindles, No. 110, or 480 power looms—in each of which cases subservient or preparatory machinery is included.

The following table will show the relative effects of different machines:

TABLE

Of the relative effects of Machines and First Movers.

Ale gallons delivered by a ten feet overshot water wheel.	Diameter in inches of a Newcomen's engine cylinder.	Diameter in inches of Watt's engine cylinder.	Number of horses working 12 hours a-day, at the rate of 2 miles an hour.	Number of men working 12 hours a-day.
230	8	6·12	1	5
390	9·5	7·8	2	10
528	10·5	8·2	3	15
660	11·5	8·8	4	20
790	12·5	9·35	5	25
970	14	10·55	6	30
1170	15·4	11·75	7	35
1350	16·8	12·8	8	40
1445	17·3	13·6	9	45
1584	18·5	14·2	10	50
1740	19·4	14·8	11	55
1900	20·2	15·2	12	60
2100	21	16·2	13	65
2300	22	17	14	70
2500	23·1	17·2	15	75
2686	23·9	18·3	16	80
2870	24·7	19	17	85
3055	25·5	19·6	18	90
3240	26·2	20·1	19	95
3420	27	20·7	20	100
3750	28·5	22·2	21	105
4000	29·8	23	22	110
4460	31·1	23·9	23	115
4850	32·4	24·7	24	120
5250	33·6	25·5	25	125

TABLE

Of the relative effects of Machines and First Movers.

Radius of Dutch sails, common position.	Best position.	Smeaton's improved sails.	Height to which the different powers would raise 1000 lbs. in one minute.
21·24	17·9	15·65	13
30·4	25·3	22·13	26
36·8	30·98	27·11	39
42·48	35·28	31·3	42
45·5	40·6	35	55
52·3	43·82	38·34	68
56·9	47·33	41·4	71
60·9	50	46·27	84
63 73	53·66	46·96	97
67·17	56·57	49·5	100
70·46	59·33	51·91	113
73·54	61·97	54·22	126
76·59	64·5	56·43	139
79·49	66·9	58·57	142
82·21	69·28	60·62	155
84·97	71·55	62·61	168
87·07	73·32	64·16	171
90·13	13·55	67·41	184
92·5	75·98	68·23	197
95	80	70	200
99·64	83·9	73·42	213
104·86	87·63	76·68	226
108·32	91·22	79·81	239
112·2	94·66	83·52	242
116·35	97·98	85·5	255

USEFUL RECIPES FOR WORKMEN.

SOLDERS.

Soldering is the art of joining two pieces of metal, by the interposition of a melted metal between them, called a solder. The solder is commonly fused by means of a heated copper bolt. We will notice the composition of the most useful solders.

For Lead—Melt one part of block tin, and when in a state of fusion, add two parts of lead. If a small quantity of this, when melted, is poured out upon the table, there will, if it be good, arise little bright stars upon it. Resin should be used with this solder.

For Tin—Take four parts of pewter, one of tin, and one of bismuth; melt them together, and run them into thin slips. Resin is also used with this solder.

For Iron—Good tough brass, with a little borax.

CEMENTS.

The first quality of all cements is tenacity in ordinary circumstances; but besides this, it is sometimes required, that they should retain this tenacity, independent of the action of heat and moisture.

A very strong glue is made by adding some powdered chalk to common glue when melted: and a glue, which will resist the action of water, may be formed by boiling one pound of common glue in two quarts (English measure) of skimmed milk.

Turkey Cement. Dissolve five or six bits of mastich, as large as peas, in as much spirit of wine as will dissolve it. In another vessel dissolve as much isinglass, (which has been previously soaked in water till it is softened and swelled,) in

one glass of strong whisky; add two small bits of gum galbanum, or ammoniacum, which must be rubbed, or ground till they are dissolved, then mix the whole, by the assistance of heat. It must be kept in a stopped phial, which should be set in hot water when the cement is to be used.

For Glass. A cement that will resist heat is composed of equal quantities of wheat flour, glass finely powdered, and powdered chalk. To this mixture, add half as much brick dust, and a little scraped lint, in the white of eggs. This mixture should be applied to the crack in the glass, and the glass should be well dried before it is put in the fire.

For turners, an excellent cement is made by melting in a pan over the fire, one pound of resin, and when melted, add a quarter of a pound of pitch—while these are boiling, add brick dust, until, by dropping a little upon a cold stone, you think it hard enough. In winter, it is sometimes found necessary to add a little tallow. By means of this cement, when warmed, a piece of wood may be fastened to the chuck, which will hold, when cool; and, when the work is finished, it may be loosed by a smart stroke with the tool.

In joining the flanches of iron cylinders or pipes, to withstand the action of boiling water and steam, great inconvenience is often felt by the workmen from want of a durable cement. The following will be found to answer:—Boiled linseed oil, litharge, and white lead, mixed up to a proper consistence, and applied to each side of a piece of flannel, linen, or even pasteboard, and then placed between the pieces, before they are brought home, as it is called, or joined.

The quantities of the ingredients may be varied, without materially hurting the cement—taking care, however, not to make it too thin by the oil, and observing that the use of the litharge is to dry speedily. This cement is useful in joining broken stones; and if the seams of the stones of a water cistern are done over with it, the durability and efficacy of the structure will be greatly promoted.

For *Steam Engines*, an excellent cement is as follows:—Take of sal ammoniac two ounces, sublimed sulpher, one ounce, and cast iron filings, or fine turnings, one pound; mix them in a mortar, and keep the powder dry. When it is to be used, mix it with twenty times its quantity of clean iron turnings, or filings, and grind the whole in a mortar, then wet it with water, until it becómes of a convenient consistence, when it is to be applied to the joint; after a time it becomes as hard and strong as any other part of the metal.

LACQUERS AND VARNISHES.

A varnish is a transparent liquid, which hardens and preserves a lustre, and is used for beautifying and preserving wood, paper, or other materials. Varnishes should be carefully kept from dust; they ought to be laid on with a brush, thinly and evenly, the strokes being all made one way, and the work done in a warm room. To remove inequalities from the surface of the varnish, pumice stone must be powdered and spread on a woollen cloth moistened with water, with which the varnish must be rubbed equally and lightly. Tripoli must then be used for polishing, the same way with olive oil, instead of water, and the whole then cleaned with a fine linen cloth and starch.

Oil Varnish is made by pouring, by little and little, half a pound of drying oil on a pound of melted copal, constantly stirring with a piece of wood. When the copal is melted, take the mixture off the fire, and add a pound of Venice turpentine. Then pass the whole through a linen cloth. When the varnish gets thick by keeping, add a little Venice turpentine; and if it be wished of a dark colour, amber should be used instead of copal.

Black varnish for iron is made of twelve parts of amber, twelve of turpentine, two of resin, two of asphaltum, and six of drying oil.

For cabinet work and musical instruments, a varnish may

be made thus:—Take four ounces of gum sandarack, two ounces of lack, the same of gum mastich, and an ounce of gum elemi; dissolve them in a quart of the best whisky; the whole being kept warm when they are dissolved, add half a gill of turpentine.

Lacquer is a varnish to be laid on metal, for the purpose of improving its appearance, or preserving its polish. The lacquer is laid on the surface of the metal with a brush; the metal must be warm, otherwise the lacquer will not spread.

For brass, a good lacquer may be made thus:—Take one ounce of turmeric root ground, and half a drachm of the best dragon's blood; put them in a pint of spirits of wine, (English measure), and place them in a moderate heat, shaking them for several days. It must then be strained through a linen cloth, and being put back into the bottle, three ounces of good seed-lack, powdered, must be added. The mixture must again be subjected to a moderate heat, and shaken frequently for several days, when it is again strained, and corked tightly, in a bottle, for use.

STAINING WOOD AND IVORY.

Yellow. Diluted nitric acid will often produce a fine yellow on wood; but sometimes it produces a brown, and if used strong it will seem nearly black.

Red. A good red may be made by an infusion of Brazil wood in stale urine, in the proportion of a pound to a gallon. This stain is to be laid on the wood boiling hot; and before it dries it should be laid over with alum water. For the same purpose, a solution of dragon's blood in spirits of wine may also be used.

Mahogany colour may be produced by a mixture of madder, Brazil wood, and log-wood, dissolved in water, and put on hot. The proportions must be varied by the artist according to the tint required.

Black. Brush the wood several times over with a hot decoction of log-wood, and then with iron lacquer; or, if this cannot be had, a strong solution of nut galls.

Ivory may be stained blue thus:—Soak the ivory in a solution of verdigris in nitrous acid, which will make it green, then dip it into a solution of pearl ash boiling hot, and it will turn blue.

To stain ivory black, the same process as for wood may be employed.

Purple may be produced by soaking the ivory in a solution of sal ammoniac into four times its weight of nitrous acid.

EXERCISES AND ADDITIONS.

It is our intention in this chapter to add to the number of examples in the text, others which may be solved by the same rules, and also others of a promiscuous nature, which require a consideration of several rules or principles. Where-ever the solution of any of these questions appears to involve any difficulty, we will give the steps of the process, and in all cases the answer.

PAGE 1—7.

Express $12\frac{7}{9}$ in the form of a common fraction. Ans. $\frac{115}{9}$

Do. $14\frac{7}{10}$ Ans. $\frac{147}{10}$

Express $\frac{12}{3}$ by the nearest equivalent whole number. A. 4.

Also $\frac{56}{7}$. Ans. 8.

Also $\frac{2918}{17}$. Ans. $179\frac{1}{17}$.

Express 9 in the form of a fraction whose denominator is 7. Ans. $\frac{63}{7}$.

Also 12 in the form of a fraction whose denominator is 13. Ans. $\frac{156}{13}$.

Express $\frac{3}{7}$ of $\frac{4}{5}$ in one fraction. Ans. $\frac{12}{35}$.

Also $\frac{2}{5}$ of $\frac{5}{8}$ of $3\frac{1}{2}$. Ans. $\frac{7}{8}$.

Express $\frac{2}{7}$ and $\frac{5}{9}$ in equivalent fractions having the same denominator. Ans. $\frac{18}{63}$, $\frac{35}{63}$.

Also $\frac{5}{6}$, $2\frac{3}{5}$ and 4. Ans. $\frac{40}{60}$, $\frac{36}{60}$, $\frac{45}{60}$.

Find the sum of $\frac{3}{5}$ and $\frac{4}{5}$. Ans. $\frac{7}{5}$ or $1\frac{2}{5}$.

Also $\frac{3}{5}$ and $\frac{5}{6}$. Ans. $\frac{43}{30}$ or $1\frac{13}{30}$.

What is the sum of $\frac{3}{5}$ and $\frac{4}{5}$, of $\frac{1}{3}$ and 9, and $\frac{3}{20}$? Ans. $10\frac{1}{60}$.

Find the difference between $\frac{5}{6}$ and $\frac{1}{6}$. Ans. $\frac{4}{6}$ or $\frac{2}{3}$.

Also $\frac{3}{4}$ and $\frac{5}{9}$. Ans. $\frac{7}{36}$.

Multiply together $\frac{3}{4}$ and $\frac{2}{9}$. Ans. $\frac{6}{36}$ or $\frac{1}{6}$.

Also $\frac{2}{3}$, $3\frac{1}{4}$, 5, $\frac{3}{4}$, $\frac{3}{5}$. Ans. $\frac{39}{8}$ or $4\frac{7}{8}$.

Find the quotient of $\frac{25}{9}$, divided by $\frac{5}{3}$. Ans. $\frac{5}{3}$, or $1\frac{2}{3}$.

It is evident that this question might have been written thus: $\dfrac{\frac{25}{9}}{\frac{5}{3}}$ or $1\frac{2}{3}$.

Also $\dfrac{3\frac{2}{5}}{4\frac{1}{2}}$ or $\dfrac{\frac{17}{5}}{\frac{9}{2}}$ or $\dfrac{34}{45}$

PAGE 7—12.

Express $\frac{5}{8}$ in decimals. Ans. ·625.

Also $\frac{3}{25}$. Ans. ·12.

Also $\frac{5}{160}$. Ans. ·03125.

Find the sum of 276, 39·213, 72014·9, 417 and 5032. Ans. 77779·113.

Also 7530, 16·201, 3·0142, 957·13, 6·72119, and 0·03014.

Find the difference of 1·9185 and 2·73. Ans. 0·8115.

Also of 2714 and ·916. Ans. 2713·084.

Find the product of ·321096 and ·2465. Ans. ·0791501640.

Also of 79·347 and 23·15. Ans. 1836·88305.

Divide ·48520998 by 178. Ans. ·00272589.

Also of 12 by ·7854. Ans. 15·278.
Also of ·8297592 by ·153. Ans. 5·4232.

PAGE 12—20.

In 35 tons, 17 cwt., 1 qr., 23 lbs., 7 oz., 13 dr., how many drams? Ans. 20571005.

How many seconds in a solar year consisting of 365 days, 5 hours, 48 minutes, 45½ seconds? Ans. 31556925·5.

Find the value of $\frac{5}{16}$ of a cwt. Ans. 1 qr. 7 lbs.

Find the value of $\frac{3}{10}$ of a day.

Express $\frac{3}{7}$ of a cwt. in the fraction of a pound. Ans. $\frac{48}{1}$.

Find the value of ·625 cwt. Ans. 2 qrs. 14 lbs.

Express in the decimal of a pound, 7 dr. avoir.

Ans. ·02734375.

Multiply 9 feet 4 inches by 3 feet 8 inches. Ans. 34 2 8.

Also 63 4 6 by 8 9 6. Ans. 557·2 0 9.

Also 365 11 8 by 13 6 3. Ans. 4948 2 11 11.

PAGE 21—24.

Find the square, cube, square root, and cube root of 531.

Ans. 281961 the square, 149721291 the cube, 23·0434372 the square root, and 8·097759 the cube root.

Find the square root of ·000729. Ans. ·027.

Find the cube root of 950. Ans. 9·830476.

Find the square, cube, square and cube roots of 872.

Ans. 760384 the square, 663054848 the cube, 29·5296461 the square root, and 9·553712 the cube root.

PAGE 24—28.

By the sliding rule,

Multiply 23 by 14. Ans. 322.

Also 27 by 23. Ans. 621.

Divide 576 by 48. Ans. 12.

Also 988 by 76. Ans. 13.
Find the square root of 576. Ans. 24.
Also of 9216. Ans. 96.

PAGE 30—32.

What length must be cut off a board that is 9 inches broad to make a square foot? Ans. 16 inches.

A wood merchant offers to sell a plank, 14 feet in length, 18 inches broad, and half a foot in thickness, at the rates of 16 pence per running foot, 14 per square, or 20 per solid foot. If he sells the plank by either of these three methods, what will he gain?

Ans. 224 running, 210 solid, and 294 pence square measure.

Sound is found to travel through the air at the rate of 370 yards in a second of time, or a mile in $4\frac{3}{4}$ seconds; but light travels so rapidly, that within all distances at which sound is heard, it may be reckoned to be instantaneous in its passage from one place to another,—so that we see the flash of a cannon the moment that it is discharged, but it is some time after when we hear the report. This circumstance, of the slowness of motion of sound compared with that of light, has been employed as a convenient measure of distance. If no other measure of time be near, the pulse may be counted, reckoning 70 pulsations in the minute.

Twelve seconds elapsed, or 840 pulsations were counted, between a flash of lightning and the hearing of the thunder; hence 70 : 840 :: 370 : $634\frac{2}{7}$ yds., or $2\frac{1}{2}$ miles, the distance of the thunder cloud.

In the night time, the flash of a gun of distress is seen, and seven seconds after, the report is heard:—determine the distance of the ship. Ans. 1·47 miles.

If a great gun is fired, on a calm day, in Glasgow green,

how long time after will the report be heard at Paisley cross, the distance being 8 miles? Ans. $38\frac{2}{5}$ seconds.

PAGE 34—37.

Find the geometrical mean between

4 and 36. Ans. 12.

144 and 576. Ans. 288.

513 and 57. Ans. 171.

128 and 32. Ans. 64.

PAGE 95—101.

Find the area of a square, each side of which is 37 feet.
Ans. 1369 sq. feet.

Find the area of a square, whose diagonal is 36 inches.

Note—diagonal $\times$ $\frac{1}{2}$ diagonal $=$ area, hence

$$36 \times \tfrac{1}{2}(36) = 36 \times 18 = 648 = \text{area.}$$

The bottom of a cistern is to be paved with stones, each of which is 2 feet 3 inches by 10 inches, and the cistern is 68 feet 3 inches long by 56 feet 8 inches broad; how many stones will be required? Ans. $2062\frac{2}{3}$ stones.

Note.—The area of the cistern's bottom is found in sq. inches; as a dividend and as a divisor the area of one stone; the quotient as the number of stones.

Find the area of a rhombus whose diagonals are 623 and 436.

Note. — The diagonals cut each other at right angles, wherefore, from the second last example,

$$623 \times \tfrac{1}{2}(436) = 623 \times 218 = 135814.$$

The base of a triangle is 396 inches, and the perpendicular 174; what is its area? Ans. 34452.

The area of a triangle may be found from the three sides, as follows:—From half the sum of the three sides subtract each side separately, then multiply continually each of these three remainders, and the half sum of the sides; the square root of this last product will be the area.

The sides of a triangle are 221, 255, and 238, then

$$\frac{221 + 255 + 238}{2} = 357;$$

and 357—the three sides successively, give remainders 102, 136, and 119, wherefore !

$$357 \times 102 \times 136 \times 119 = 589324176,$$

the square root of which is 24276 = the area.

In a trapezoid, the parallel sides are 96 and 143, the perpendicular distance between them being 89, find the area.

$$(143 + 96) \times \tfrac{1}{2}(96) = 239 \times 44{\cdot}5 = 10635{\cdot}5.$$

What is the area of a regular heptagon, or seven-sided polygon, one of whose sides is 327 inches?

Ans. 388570·6190196.

Find the circumference of a circle of which the diameter is 628 inches. Ans. 1972·9248.

Also the circumference of a circle whose radius is 3 feet 2 inches. Ans. 238·7616.

Find the diameter of a circle whose circumference is 984 inches. Ans. 313·21693.

Also the radius of a circle whose circumference is 24855·43 inches. Ans. 3955·96.

Find the area of a circle whose diameter is 10 and circumference 31·416. Ans. 78·54.

Find the area of a circle of which the radius is 6 feet 6 inches, or 78 inches.

Ans. $78^2 \times 3{\cdot}1416 = 19113{\cdot}4944$ sq. inches,

or $78 \times 2 =$ diameter $= 156$, hence $156^2 \times {\cdot}7854 = 19113{\cdot}494$,

$$\text{or } \frac{156^2 \times 3{\cdot}1416}{4} = 19113{\cdot}494.$$

Find the area of a circle of which the circumference is 1284 inches.

$$\frac{\text{Half the circumference}^2}{3{\cdot}1416} = \text{area},$$

hence $$\frac{642^2}{3{\cdot}1416} = 131195{\cdot}569$$

What is the area of a circle whose diameter is two inches?

Ans. 3·1416.

Note. — Hence, a circular inch is to a square inch, as 3·1416 is to 4.

If the area of a circle is 62 inches, then the area of the circumscribing square will be 3·1416 : 4 : : 62 : 78 and a fraction.

There is a ring whose outer diameter is 10, and inner 6 inches; what is the area of the flat face of the ring?

$$(10 + 6) \times (10-6) \times {\cdot}7854 = 50{\cdot}2656 = \text{area.}$$

Find the area of an ellipse, whose axes are 7 and 5.

Ans. 27·489.

Find the area of a parabola, whose base is 54 and height 36 inches. Ans. 1296.

Find the surface of lead necessary to form a cistern, whose length is 7 feet 8 inches, breadth 4 feet 7 inches, and depth 2 feet 9 inches. Ans. 137 feet, 7 inches, 10 pts.

Find the solid content of the earth, supposing its diameter to be 7912 miles, it being a sphere.

Ans. 259332805349·95 cubic miles.

How many square feet of deal will be required to make a chest, whose length is 3½ feet, breadth 2 feet, and depth 20 inches? Ans. 32⅓ square feet.

How many 3 inch cubes can be cut out of a 12 inch cube?

Ans. 64.

Four men bought a grindstone, which was 30 inches in diameter, and agreed that each should work one fourth of it, exclusive of 6 inches diameter at the centre, which was allowed for waste. They used the stone, one after the other —what part of the diameter did each grind down?

Note.—The solution of this will evidently depend on dividing the grindstone into 4 = zones, exclusive of the waste part. Now the area of the whole face of the stone is $30^2 \times {\cdot}7854 = 706{\cdot}86$, the area of the waste part is $6^2 \times 7854 = 28{\cdot}2744$; therefore the zone to be divided is

706·86—28·2744=678·5856, the fourth of which is 169·6464, and as circles are to each other as the squares of their diameters, therefore we have
706·86 : (706·86—169·6464) = 537·2136 :: 30^2 is to the diameter square of the stone, after the first has taken away his share, which is 684·8 nearly, whose square root is nearly 26·1725, therefore 30—26·1725 = 3·8275 as the part of the diameter ground away by the first; and in the same way the others may be found to be 4·5201, 5·7588, and 9·8745 inches.

What length of wire, $\frac{1}{40}$ of an inch in diameter, can be drawn out of a cubic foot of brass, no metal being lost, $\frac{1}{40}$ = ·025, hence $·025^2 \times ·7854$ = ·000490875, and every time this is repeated in 144, there will be a foot in length of the wire? Ans. 56 miles nearly.

PROMISCUOUS.

A body weighing 20 lbs. is impelled by a force which causes it to move 100 feet per second; if the body had been 8 pounds weight, what would have been its velocity, impelled by the same force? Ans. 250 feet per second.

The battering ram of Vespasian weighed 10000 lbs., and was urged against the walls of Jerusalem, with a velocity of 20 feet per second. By this means the walls were demolished. Find the velocity that must be given to a 32 lb. cannon ball, to do the same? Ans. 6250 feet.

Divide the beam of a steel-yard, so that the weights 1, 2, 3, 4, &c., lbs. on the one side, will balance a constant weight of 95 lbs., at 2 inches from the fulcrum, on the other side, the whole length of the beam being 3 feet, and its weight 10 lbs.

The weight of the short arm will be found $=\frac{2 \times 10}{36}=$ ·55,

and the length of the long arm being 34, its weight will be found = 9·45.—The momentum of the weight at short arm, and the weight of that, are 95 × 2 + ·55 = 190·55. The momentum of long arm will be found = 160·65, wherefore the difference of the momentums will be 29·90, or in round numbers 30, hence 30, divided by 1, 2, 3, 4, &c., will give 30, 15, 10, 7½, 6, 5, 4$\frac{2}{7}$, 3¾, &c., for the respective distances of the weights.

Find the specific gravity of a stone which weighs 7 lbs. in air, and 5 lbs. in water.

$$\frac{7 \times 1000}{7 - 5} = 3500.$$

A piece of copper weighs 18 lbs. in air, and 16 lbs. in water, and there is affixed to the copper a piece of elm wood, whose weight is 15 lbs. in air, and the weight of the two in water is 6 lbs. Find the specific gravity of the elm.

$$\frac{15 \times 1000}{16 - 6 + 15} = 600 = \text{Ans.}$$

Note.—The whole weight of a body which will float in a fluid is equal to the weight of as much of the fluid as the immersed, part of the body displaces when it floats. Also the magnitude of the whole body is to the magnitude of the part immersed as the specific gravity of the fluid is to that of the body.

Find the depth to which a cubic foot of oak will sink in common water, its specific gravity being 925, and that of water 1000. Ans. 11·1.

A very large steam vessel is let down into a basin which is brim-full of water, the vessel being allowed to float, is found to cause an overflow of 50000 cubic feet of water, from which circumstance find the whole weight of the vessel.

Ans. 1365·1 tons.

Hiero, king of Sicily, ordered his jeweller to make him a crown of gold, containing 63 ounces. The crown being made, the king suspected it was not pure, and set Archime-

des to examine it, who found that, when put into water, it displaced 8·2245 cubic inches. On weighing an ounce of pure gold, he found it 10·36 ounces, and silver 5·85 ounces; from which circumstance, he found that the crown was not pure gold, but alloyed with silver. Find the proportion of each. Ans. 28·8 of silver.

Find the power of a fall of water, whose height is 30 feet, velocity 20 feet per minute, and suction at the bottom 10 feet. Ans. 6000 lbs.

Find the number of horses' power for an undershot wheel, driven by the above fall, and also that of an overshot, together with the quantities of wheat ground by each, allowing one imperial quarter of wheat for each horse power in an hour—the mill working 12 hours a day.

$$\frac{6000}{5000} = 1{\cdot}2 \text{ horse's power for an undershot,}$$

$$\frac{6000}{5000} \times 2{\cdot}5 = 3 \text{ horses' power for an overshot,}$$

and $1{\cdot}2 \times 1 \times 12 = 14{\cdot}4$ imperial quarters of wheat ground by the undershot in a day, and 36 by the overshot.

Find the diameter of a baloon which will rise with 340 lbs., including its own weight, the specific quantity of hydrogen gas being 0·1, and the common atmospheric air at the surface of the earth 1·2.

The difference of the weight of a cubic foot of gas and air is 1·1 ounce, hence 340 lbs. reduced to ounces = 5440, wherefore 1·1 : 5440 :: 1 : 494 = the content of a sphere which will balance a weight of 340 lbs., and by the principles of geometry, and the rules of mensuration, and it may be found, that the content of a sphere, whose diameter is 3 feet 6 inches, is 22·449 cubic feet; and spheres being to each other as the cubes of their diameters, we have in round numbers $22{\cdot}5 : 494 :: 3{\cdot}5^3 : 1495$, the cube root of which is 11·11 feet the diameter of baloon. Any greater diameter than this will cause the baloon to rise.

A stone being let fall into a coal-pit takes 12 seconds to reach the bottom: find the depth of the pit. Ans. 2316 feet.

A beam, 16 feet long, resting on two props, bears a weight of 6 tons at the distance of 3 feet from one end, and a weight of 4 tons 2 feet from the other end. Find the weight borne by each prop. Ans. $4\frac{10}{16}$ and $5\frac{6}{16}$.

Being desirous to measure the height of a tower, a stone was tied to a string 18 inches long, thus forming a kind of pendulum; a stone was then let fall from the top to the bottom of the tower, and during the time of its descent, the string pendulum made 8 vibrations. Find the height of the tower.

Hence $\left(\frac{18}{39 \cdot 1386}\right)^{\frac{1}{2}}$ = the time of one vibration of the pendulum 18 times, which will give the time that the body takes to fall, which will correspond to the space 412·61 feet.

A power of 20 lbs. is applied to the winch of a crane which is 10 inches in length, the pinion makes 15 revolutions for one of the wheel, and the barrel on which the chain is coiled is 8 inches diameter; find the power of the crane.

Here we must find, in the first place, the circumference of the circle which the handle of the winch describes =

$$3 \cdot 1416 \times 20 = 62 \cdot 836,$$

and as the pinion, and consequently the winch, makes 15 turns for 1 of the barrel, we have

$$62 \cdot 836 \times 15 = 942 \cdot 54 =$$

the space through which the power moves during one revolution of the barrel—and

$$942 \cdot 54 \times 20 = 18850 \cdot 8 =$$

the power $\times$ the distance it travels—and

$$8 \times 3 \cdot 1416 = 25 \cdot 1328 =$$

the distance the weight travels, hence

$$\frac{18850 \cdot 8}{25 \cdot 1328} = 750 \text{ lbs.}$$

with a fraction, the weight which can be raised by the crane with a power of 20 lbs.

NUMBERS USEFUL IN CALCULATION.

If the diameter of a circle,	= 1
Circumference,	= 3·1415926
Area,	= ·7853981
Side of inscribed equilat. triangle,	= ·8660250
Circumscribed do.	= 1·7320508
Inscribed Square,	= ·7071067
Side of equilat. triangle,	= 1·
Diameter of circumscribed circle,	= 1·1547
Diameter of inscribed do.	= ·5773
Side of a square,	= 1
Diameter of a circumscribed circle,	= 1·414213

The pressure of the atmos., = 15 lbs. to the square inch,
= 30 inches of mercury,
= 32 feet of water,
= the elastic force of steam at 212.

Length of a second's pendulum $39\frac{1}{8}$ inches.

Cohesion of a cubic inch of cast iron = 18000 lbs.

A body will fall freely in the first second $16\frac{1}{12}$ feet.

THE END.

ERRATA.

Owing to the rapidity with which this work has been sent through the Press, a few errors have occurred, which the reader is desired to correct.

Page 8, lines 7 and 8—*for* ninety *read* unity.

—— 21, line 12—*for* 115 *read* 125.

—— 95, line 16—*for* plane *read* prism.

—— 108, line 10 from bottom—*for* $\frac{(160)^{\frac{1}{2}}}{32}$ *read* $\frac{160}{32}$.

—— 115—The steel yard, a weight of 2 ounces, at the distance of 1 inch will be balanced by 1 ounce at 2 inches, &c.

—— 172, bottom line—*for* 1 to 2·832 *read* 3·1416 to 4, and the 4287 in the example refers to lbs., a whole number.

—— 190, line 5—transpose stream and wheel.

—— 192, line 3—See page 197, half way down.